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AND

DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

TRUCK SPEED AND TIME LOSS ON GRADES

By J. W. Stevens,[1] Assoc. M. ASCE

Synopsis

In the design of grades it is frequently necessary to ascertain the delay to traffic resulting from the decrease in speed of heavily loaded trucks on adverse grades. On routes involving heavy traffic it is particularly important to determine the effect of speed reduction and time loss incurred by loaded trucks on hill and overpass grades upon the operation of traffic at a constant design speed. Where the time loss is large it may be necessary to use preventive or remedial measures, such as an alternate route in rough terrain, a reduction of the rise in grade at an overpass, or in an extreme case the construction of an additional traffic lane, in order to avoid congestion for the anticipated traffic density.

A procedure for calculating the velocity curve and determining the time lost by a given vehicle traveling over an adverse grade is described in this paper. It is possible to determine the relative advantages of alternate grade designs with regard to the operation of truck traffic. The method outlined was developed in the design of a freeway project, and the examples used in the paper are concerned with overpass grades on that project. However, the application of the method is wider than that covered by the examples. The entire range of problems involving time loss on grades can be handled by the outlined methods, modified as suggested in the paper.

In the freeway design problem, certain conclusions were reached concerning the effect of variations in grade rise and grade rate at overpasses upon time loss. These conclusions and some general comments regarding the effect of truck speed reduction upon highway traffic capacity are included in the paper.

1. Basic Data and Principles

Basic data for computing truck velocities on grades were published in 1942 by Carl C. Saal,[2] including tests made by the U. S. Public Roads Administration

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by February 1, 1947.

[1] Bldg. Insp. Engr., City of Amarillo, Amarillo, Tex.

[2] "Hill-Climbing Ability of Motor Trucks," by Carl C. Saal, *Public Roads*, May, 1942, p. 33.

(USPRA) to determine the velocities that could be maintained on various grades by trucks carrying various loads, and to determine the decrease in performance of trucks due to age. Although the direct conclusions to be drawn from Mr. Saal's findings pertain to the maximum sustained speed of loaded trucks on long grades, the tests also included the determination of various data which are required for the computation of truck speeds on short grades and on combinations of up-and-down grades under different conditions of loading and power.

In the calculation of velocity and time curves for trucks, operating over a given profile, it is possible to utilize principles which have been used for some time in studies of railroad train operation. Since the operating characteristics of motor vehicles are considerably more flexible than those of railway trains, it is probable that the results of calculations for trucks are subject to errors which would not be encountered in railroad work. Also, since experimental data on the tractive resistance of trucks are scarce, it will often be necessary to use assumed values for this factor. Furthermore, the difference in driving habits of vehicle operators and the total lack of knowledge regarding what may be expected as an average procedure for the application of the available tractive effort of the vehicle in question necessitate the use of rather broad assumptions relative to the manner of applying the motor power, especially on descending grades. It is desirable that additional test data on tractive resistance and driver habits be obtained, in order that the limitations which these features impose on velocity-curve determination will be removed.

2. Fundamental Equations of Motor Vehicle Dynamics

The forces that affect the speed of a motor vehicle may be grouped under three main classifications: (1) The tractive effort exerted by the motor through the driving wheels; (2) the total tractive resistance of the vehicle, including rolling and air resistance; and (3) the accelerating or decelerating force created by the grade on which the vehicle is operating. The first two classes of forces are subject to determination by more or less elaborate tests; the third is easily computed from the gross weight of the vehicle and the rate of grade.

The familiar equation, $F = m\,a$, with modifications for the problem under consideration, is used for calculating velocity changes with varying forces. Mr. Saal uses the equation:[3]

$$P_e = a\,(m + k) + W_g\,(f) \qquad (1)$$

in which P_e is the net tractive effort, in pounds, at drive wheels; a is the acceleration of vehicle, in feet per second per second; m is the mass of vehicle $= W_g/32.2$, in pounds mass; k is the mass equivalent constant for rotating parts of vehicle, in pounds mass; W_g is the gross vehicle weight, in pounds; and f is the coefficient of tractive resistance, in pounds per pound.

Eq. 1 is applicable for level grade, and it may be modified to the following form, which is applicable, with only small error, for inclined grades:

$$P_e = a\,(m + k) + W_g\,(f) + W_g\,(S) \qquad (2)$$

in which S is the rate of grade, in feet of rise per foot of horizontal distance.

[3] "Hill-Climbing Ability of Motor Trucks," by Carl C. Saal, *Public Roads*, May, 1942, p. 49, Eq. 15.

Eq. 2 may be rearranged in the following form:

$$a = \frac{P_e - [W_g(f) + W_g(S)]}{m + k} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

and the acceleration a thus evaluated may be used in calculating the change in velocity of the vehicle in a given distance or time interval, when the initial velocity is known.

3. Use of Numerical Integration for Calculating Velocity Curves

The method described herein for computing velocity curves for motor vehicles involves the use of a numerical integration process, which is based on the following principles:

Fig. 1 shows velocity, acceleration, and time curves, referred to distance plotted along the abscissa. If the velocity v_1 is known, an approximation to v_2 may be obtained by means of the relation,

$$v_2 = v_1 + a_m (t_2 - t_1) \quad \ldots\ldots (4)$$

in which a_m is the mean value of the acceleration between s_1 and s_2. It is evident that the time interval, $(t_2 - t_1)$, is equal to the distance interval, $(s_2 - s_1)$, divided by the mean velocity between s_1 and s_2. If sufficiently small intervals of distance are used, any desired degree of accuracy may be obtained in the approximation; the arithmetic mean values, $(v_1 + v_2)/2$ and $(a_1 + a_2)/2$, may be used, instead of the actual mean values, for velocity and acceleration, respectively, obtained by dividing the areas under the corresponding curves in the interval (s_1 to s_2) by the length of the interval.

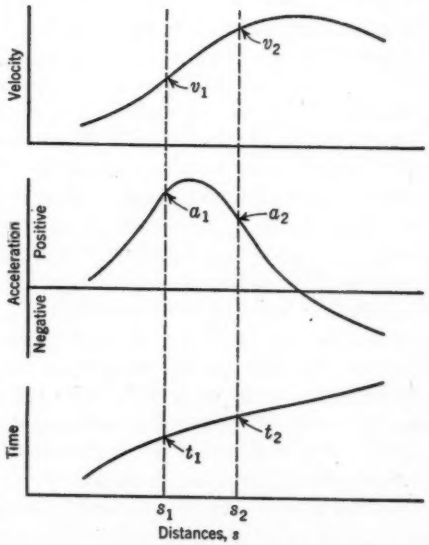

FIG. 1.—CHARACTERISTIC VELOCITY, ACCELERATION, AND TIME CURVES

4. Outline of Method for Computing Velocity Curves

The method for computing the velocities of a given vehicle at successive points along a given profile may be stated briefly as follows:

Uniform distance intervals, sufficiently short to afford the desired degree of accuracy in velocity and time values, are marked off on the profile throughout the section under consideration. The uniform intervals may be subdivided if necessary at points where one or more of the forces acting on the vehicle changes abruptly. The change in velocity in each distance interval is calculated by the procedure outlined in the next paragraph; and the velocity at the end of each interval, the mean velocity in the interval, and the time required to traverse

the interval are computed. From the series of values obtained for the successive intervals through the section of profile being studied, the velocity curve may be plotted, and the time lost by the vehicle, compared to the time required for traveling over the section at a given uniform velocity, may be determined.

The forces acting on the vehicle at a given point, such as the beginning or the end of a distance interval, are determined by the power developed by the motor, the rate of grade at the given point, and the velocity of the vehicle. These forces are the tractive effort P_e in Eq. 3 which may be computed from the motor power by a method explained in Section 7, the grade resistance W_g (S) in Eq. 3, and the tractive resistance W_g (f) in Eq. 3 (which varies with the vehicle velocity). The acceleration or deceleration of the vehicle at the beginning of the distance interval under consideration is computed by means of Eq. 3; that is, by dividing the resultant force (which is the algebraic sum of the three forces, P_e, W_g (S), and W_g (f)) by the sum of the mass (m) of the vehicle and a mass equivalent constant (k) for the rotating parts of the vehicle. In a similar manner the forces acting on the vehicle and the acceleration or deceleration at the end of the distance interval are computed. Since the velocity at the end of the interval is unknown at this stage, an assumed value of f must be used to obtain the tractive resistance. The use of the value of f corresponding to the velocity at the beginning of the interval in question will afford satisfactory accuracy; although, if desired, the value of f corresponding to the approximate anticipated velocity at the end of the interval may be used for closer results. The average value for acceleration or deceleration in the distance interval is computed by taking the arithmetic mean of the values at the beginning and the end of the interval. Then the change in velocity through the distance interval is calculated by multiplying the average acceleration or deceleration by an assumed period of time for the vehicle to traverse the distance interval under consideration. This change in velocity is added algebraically to the initial velocity for the interval to obtain the velocity at the end of the interval, and the arithmetic mean velocity for the interval is computed. The time required for traversing the interval at this average velocity is computed by dividing distance by velocity. If the time thus obtained does not check that previously assumed, the computation of velocity change and the succeeding calculations are repeated until corresponding values for time and mean velocity are obtained. Fair accuracy usually may be secured on the first trial, since the time can be estimated closely from the initial velocity, and from a knowledge of whether the acceleration is positive or negative. When satisfactory agreement between time and mean velocity is obtained, the corresponding values for these items and for the velocity at the end of the interval are taken as final for the interval under consideration. The foregoing series of operations are repeated for the successive distance intervals.

5. Vehicle Data Required for the Computations

The vehicle data required for the computations consist of the tractive effort available at the driving wheels, the tractive resistance of the vehicle for the entire range of velocities involved in the problem, the mass of the vehicle, and the mass equivalent constant for the inertia of the wheels and other rotating

parts of the vehicle. Fairly complete data for the trucks and truck-trailer combinations tested by the USPRA are given by Mr. Saal.[2] The tractive resistance of a vehicle varies considerably for different types of pavement or road surface. The values which he gives are applicable for concrete pavement, and should be used only for concrete and closely similar types of surfacing. The tractive-resistance data presented by Mr. Saal are applicable to the low-type bodies used in the tests, and higher values should be used for large van-type bodies.

6. Application of Available Tractive Force

There is great uncertainty as to the manner in which the available tractive force of a vehicle will be applied by an individual driver; and there are little or no observed data to indicate an average habit pattern for the application of the tractive force. Therefore, it is necessary to use a driving procedure, for velocity-curve computations, based almost entirely upon assumption. For the 15-ton truck and the type of profile considered in this study, involving relatively low rises in grade in level terrain, it was decided to consider operation in high gear only. For a heavily loaded truck on ascending grade, it appears entirely logical to assume that the tractive force applied will be at or near the maximum available value, as long as the speed of the vehicle is decreasing. Such an assumption was used in the various cases covered in this study. It was also assumed that the tractive effort is increased uniformly from the value required for constant initial speed on level grade to the maximum available value as the vehicle traverses the sag vertical curve at the beginning of the grade rise. Thus it is assumed that other traffic prevents the operator of a heavily loaded truck rom increasing speed prior to reaching the rising grade to compensate for the expected loss in speed, and that full power is applied by the time the maximum rate of grade is encountered.

It is more difficult to state what procedure can logically be assumed for application of the available motor power on a descending grade, or even after the vehicle has begun to accelerate in passing over a summit at full throttle. The most serious limitation upon the accuracy of computed velocity curves results from this feature. On a descending grade the vehicle may be operated in a great variety of ways—with the throttle fully open, partly open, or closed, in gear, or in neutral, and with or without application of the brakes. Any combination of these operating conditions in succession may be encountered. It is evident that very complicated tests would be required in order to determine, under such a variety of possibilities, a typical or average driving pattern of procedure. Lacking such test data, one can only attempt a fairly logical assumption for driving procedure on descending grades.

In the cases for which velocity curves were computed in this study, it is assumed that the truck is moving on a level grade at a speed of $V = 40$ miles per hr prior to entering the grade up to an overpass; and that the tractive effort is uniformly increased from the value required for operation on level grade to the maximum available value as the vehicle moves through the vertical curve from level to ascending tangent grade. The maximum tractive effort is maintained until the vehicle has begun to accelerate near the summit of the grade

rise. Then it is gradually reduced to zero and maintained there until the truck is near the end of the descending grade, when it is gradually increased to the value for operation on level grade at the initial speed of 40 miles per hr. It is the intention to apply the power or tractive effort in accordance with the foregoing procedure in such a way that the vehicle speed will be restored to exactly 40 miles per hr at the end of the descending grade. This procedure does not result in the minimum possible time loss on the grades, for it does not take account of a possible increase in speed to a value greater than 40 miles per hr on the descending grade. Operation on the descending grade at a speed greater than 40 miles per hr might require the use of the brakes to avoid running too close to the vehicle ahead of the truck. In such a case, a time gain rather than a time loss is involved. Operation over the profile with minimum time loss could be effected only by the most skilful drivers, and certainly not by the average driver. The assumptions used in the problems are chosen to represent average conditions.

The objective of restoring the speed to 40 miles per hr at the end of the descending grade can be attained only by a trial procedure; but it is possible to use the acceleration curve as a basis for assumptions regarding the use of the tractive effort, so that this curve encloses approximately equal areas above and below the zero axis, within the limits of the rising and descending grades. In other words, the speed at the end of the overpass grade will be restored to the original value if the product of the deceleration and the distance through which it operates is made approximately equal to the product of the acceleration and the distance through which it operates. This technique is subject to a minor error resulting from the fact that the acceleration curve is plotted to a distance abscissa, rather than to a time abscissa.

For the grade problems discussed herein, it is assumed that the vehicle remains in high gear at all times when power is applied. This assumption is legitimate for the range of speeds involved, since about 30 miles per hr is the minimum speed encountered in normal overpass grade problems. On a given grade, the characteristics of the truck and the habits of the driver will determine the speed at which gear changes will be made. It is considered very unlikely that a change from high gear to second would be made under ordinary circumstances at a speed above 30 miles per hr. For the grade conditions considered herein, it appears probable that operation in high gear will result in a minimum time loss, since the time and speed lost in shifting gears twice would most likely counteract any possible saving in time realized by the utilization of the higher tractive effort available at a higher gear ratio.

In problems concerning long steep grades, of course, it will be necessary to consider changes in gear ratio in computing the velocity curves. This feature affects the tractive-force diagram and the mass equivalent constant for rotating parts of the vehicle, but it does not introduce serious complications in the calculations.

7. Formulas for Maximum Tractive Effort

The maximum tractive effort available at the drive wheels for a given vehicle may be calculated for various engine speeds and gear ratios, from data

relative to the engine horsepower or torque and the over-all mechanical efficiency of the transmission and the rear end drive mechanism. For conversion of truck speed to engine speed, or vice versa, the following formula[4] is applicable:

$$\omega = \frac{168\ G\ V}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which ω is the engine speed, in revolutions per minute; G is the ratio of total gear reduction; V is the road speed, in miles per hour; and r is the effective radius of drive wheels, in inches.

When the net engine horsepower for a given engine speed for full throttle operation is known, the net engine torque may be computed by means of the following relation:

$$T = \frac{33{,}000\ E}{2\ \pi\ \omega} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which T is the net engine torque, in pound-feet; and E is the net engine horsepower. If the net engine torque curve for a given truck is available, the tractive-effort curve for a given gear ratio may be derived from it by using the following formula (given in slightly different form[5] by Mr. Saal):

$$P_e = \frac{12\ T\ G\ \eta}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which P_e is the net tractive effort, in pounds, available at drive wheels, corresponding to a given engine speed, at full throttle; and η is the ratio of over-all efficiency of drive mechanism, for given speed and gear ratio.

Substitution of the expression for torque, Eq. 6, in Eq. 7 gives the following formula for tractive effort:

$$P_e = \frac{12\ E \times 33{,}000\ G\ \eta}{2\ \pi\ r\ \omega} = \frac{198{,}000\ E\ G\ \eta}{\pi\ r\ \omega} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

8. Performance Data for Truck-Trailer Combination

The truck-trailer combination selected for example purposes in the computed velocity curves included in this study is a medium truck and a high van-type semi-trailer, having a gross weight (W_g) of 30,000 lb. Average performance data, rather than actual data for any one make and model of truck, are used. The maximum net horsepower output of the motor (E) is taken at 90 at a speed (ω) of 2,800 revolutions per min, and the over-all efficiency of the drive mechanism (η), exclusive of the motor, is assumed to be 90% throughout the range of speeds involved. Actually, the efficiency factor for a given gear ratio varies slightly with speed; however, sufficiently accurate results for the purpose of this study will be obtained by the use of a constant efficiency factor. Also, the tractive effort available at full throttle at a given gear ratio varies with the speed, but it was considered satisfactory to use a constant value for this factor, for the range of speeds involved. Using Eqs. 5 and 8, with a total gear reduction (G) of 7.15 and an effective drive wheel radius (r) of 18.5 in.,

[4] "Hill-Climbing Ability of Motor Trucks," by Carl C. Saal, *Public Roads*, May, 1942, p. 45, Eq. 10.
[5] *Ibid.*, p. 49, Eq. 16.

the tractive effort (P_e) available at the wheels for full throttle operation was found to be 750 lb for a truck speed (V) of 40 miles per hr and 780 lb for a speed of 30 miles per hr. These values correspond to horsepower values (E) of 89.0 and 69.3, respectively, taken from a typical power curve for a 90-hp engine. A maximum tractive-effort value of 760 lb is used in the computations.

Values for the mass equivalent constant (k) used to represent the energy stored in rotating parts of the truck and trailer combinations were determined experimentally and are shown, in charts, by Mr. Saal.[6] For the truck and trailer type considered in this study, operating in high gear at a gear ratio of 7.15, Mr. Saal indicates that a mass equivalent constant of about 70 is applicable. The total mass of the vehicle effective in resisting acceleration or deceleration is computed as follows: Total mass $= m + k = \frac{W_g}{32.2} + 70$ $= \frac{30{,}000}{32.2} + 70 = 1{,}002$. To simplify the computations in this case, it is sufficiently accurate to use a value of 68 for k, which makes the total mass equal to 1,000 lb.

Two tractive-resistance curves are shown in Fig. 2. These curves represent data for the average tractive resistance of medium tractor trucks with low-bed semi-trailers, operating on concrete pavement in very good condition.[7] These curves represent the data for a combination with gross loads of 30,000 lb and 20,000 lb, as shown. In the computations of Section 11 the tractive resistances used are based on the 20,000-lb curve, on the assumption that it is approximately correct for the van type of bed with a load of 30,000 lb, since the air resistance will be higher than that for a low bed.

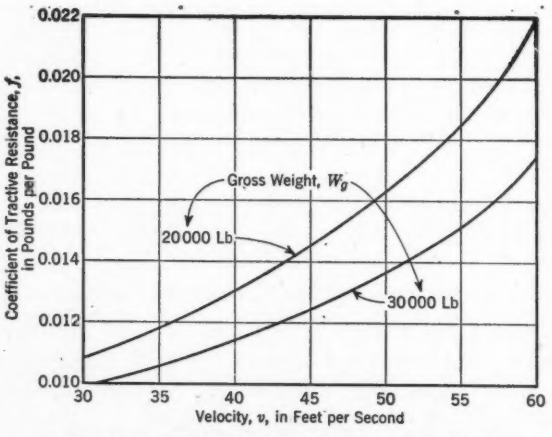

FIG. 2.—TRACTIVE-RESISTANCE CURVES, LOW-BED TRAILER ON CONCRETE PAVEMENT

The foregoing data constitute all the information relative to the vehicle required for computation of the velocity curve. Information regarding the profile is necessary to complete the data for the calculations.

9. DATA RELATIVE TO PROFILE

In the present study, made in connection with a divided highway in an urban area, where the average speed will be about 40 miles per hr, it is con-

[6] "Hill-Climbing Ability of Motor Trucks," by Carl C. Saal, *Public Roads*, May, 1942, p. 47, Figs. 17 and 18.

[7] *Ibid.*, p. 43, Table 15.

sidered that a vertical sight distance of 500 ft should be provided, based on a line of sight 4.5 ft above the pavement surface at each end. This design requirement governs the length of summit vertical curves for various rates of grade on each side of the summit. The grade lines used in the speed-loss computations for rises of various heights were made symmetrical about their high point, the midpoint of the summit vertical curve, except in the case involving a rise without an equal grade drop. The lengths of the short vertical curves connecting to level grade at each end of the grades over the overpass were chosen in most cases so that a grade change of 5% per station results. For a given rise in grade above a level line there is a certain maximum rate of grade that can be used under the aforementioned conditions. This maximum grade will be that for which the ends of the summit vertical curve coincide with the ends of the sag vertical curves connecting to the level grade. Generally, this maximum rate of grade will provide the most economical solution from the standpoint of construction cost. There is a possibility, however, that it may involve excessive delay to traffic, on account of steep grades, if a heavy volume of truck traffic is involved. It might even be excessive for any type of traffic.

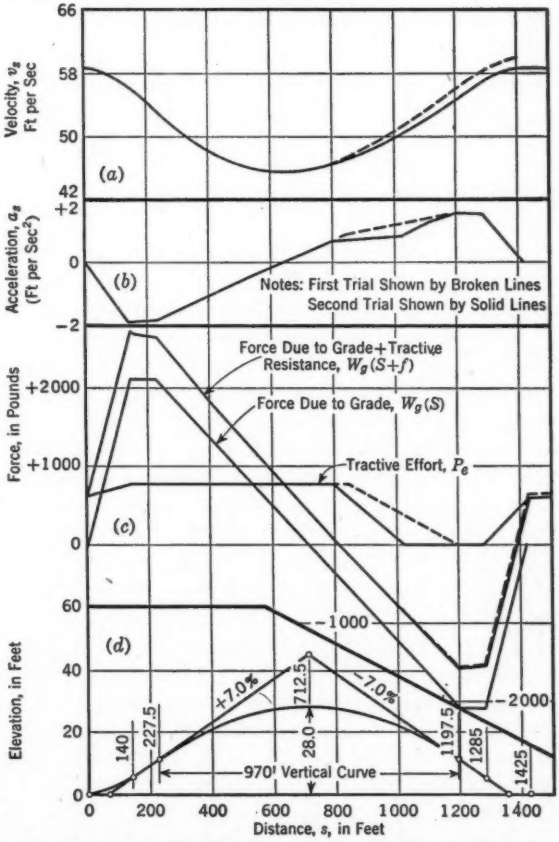

FIG. 3.—FORCE, ACCELERATION, AND VELOCITY CURVES (GROSS WEIGHT, 30,000 LB; 90-HP MOTOR; HIGH GEAR)

The grade line for the illustrative problem, for which detailed computations are outlined, provides a rise of 28 ft above the level approach grades, which is approximately the height required for a railroad overpass. A rate of grade of 7.0% was chosen as being about the maximum that is satisfactory for the type of highway under consideration. The length of vertical curve required to provide a sight distance of 500 ft between the +7.0% and −7.0% grades is 970 ft. The length of sag vertical curves required to connect the 7.0% grade

and the level grade is 7.0/5.0 = 1.4 stations or 140 ft. The essential features of the profile are now known, and the grade line is plotted in Fig. 3, for convenience in visualizing the problem in the computations of Section 11.

10. Force Diagrams and Acceleration Curve

The next step is to plot the force diagram for the given profile. The force created by the grade, acting to change the velocity of the vehicle, is the product of the gross vehicle weight and the rise or fall of the grade in feet per foot of distance measured along the roadway surface. It is sufficiently accurate and more convenient to use the grade ratio of feet of rise or fall per foot of horizontal distance in computing the force due to grade. The error involved for a 7.0% grade is slightly less than 0.25%, which is negligible.

Where the grade is tangent, the force due to grade is constant. On vertical curves the rate of grade, and as a consequence, the force due to grade, vary uniformly. Since the force due to grade is determined by the profile, the complete diagram for this force may be drawn at once; but the diagrams for the tractive effort exerted by the truck and the tractive resistance cannot be computed at this stage of the work. The diagram for tractive resistance can only be plotted as the velocity computations are made, because the resistance varies with the speed of the vehicle. The curve for tractive effort is projected by trial, in an attempt to provide for the restoration of the vehicle speed to the initial value of 40 miles per hr at the end of the descending grade. In Fig. 3(*c*) the tractive-effort diagram is plotted upward from the zero force axis. The diagram for force due to grade is plotted upward for plus grades and downward for minus grades; and the tractive resistance is plotted upward from the grade-force diagram, as the successive tractive-resistance values are determined. This method of plotting provides a curve representing the resultant of the external forces acting on the vehicle—that is, all the forces except the tractive effort. The difference between the ordinate to this resultant curve and the ordinate to the tractive-effort curve, at a given point on the profile, is the net force acting to accelerate or decelerate the vehicle. When the resultant curve is above the tractive-effort curve, the computed net force is negative, decelerating the vehicle; when the resultant curve is below the tractive-effort curve, the net force is positive, and the vehicle is accelerating. Furthermore, the ordinates to the acceleration curve are proportional to the corresponding differences between the ordinates to the resultant curve and the tractive-effort curve. With this arrangement, it is easy to visualize the effect of changes in the applied tractive effort. By studying the acceleration curve from time to time after the vehicle is on the descending grade, the tractive-effort curve may be projected in such a manner that fewer trials will be required to attain the desired vehicle speed at the end of the grade.

11. Description of Detailed Computations

With the data in Sections 8 and 9, the numerical computations can be arranged in the form shown in Table 1, or in any shorter form suitable to the computer. The normal interval of distance is 50 ft, but, wherever the force

acting on the vehicle changes sharply inside the normal 50-ft interval, the normal interval is subdivided. For instance, the normal interval is divided at the end of the sag vertical curve, distance $s = 140$, where the force due to grade reaches a maximum. The use of short intervals in this manner improves the accuracy of the approximation.

For the profile under consideration, the grade is level at distance $s = 0$ and the grade resistance, therefore, is zero. The velocity of the truck is 40 miles per hr or 58.667 ft per sec at distance $s = 0$. The coefficient of tractive resistance (f), taken from the curve in Fig. 2, is 0.0206 lb per lb of truck weight. The tractive resistance (R_t) corresponding to the initial velocity, and used also for

TABLE 1.—COMPUTATIONS FOR THE VELOCITY CURVE

FEET	POUNDS				FEET PER SEC²		FEET PER SEC		LB PER LB	FT PER SEC	SECONDS	
s	P_e	R_t	R_g	P_n	a_e	a_n	Δv_e	v_e	f	V	Δt	t
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(a) DECELERATION ON RISING GRADE; MAXIMUM P_e APPLIED												
0	+618	−618	0	0	0	....	....	58.667	0 0206	.....	....	0
50	+669	−618	−750	−699	−0.699	−0.350	−0.299	58.368	0.0205	58.518	0.854	0.854
100	+719	−615	−1,500	−1,396	−1.396	−1.048	−0.904	57.464	0.0198	57.916	0.863	1.717
140	+760	−594	−2,100	−1,934	−1.934	−1.665	−1.171	56.293	0.0193	56.879	0.703	2.420
150	+760	−579	−2,100	−1,919	−1.919	−1.926	−0.343	55.950	0.0190	56.122	0.178	2.598
200	+760	−570	−2,100	−1,910	−1.910	−1.914	−1.738	54.212	0.0181	55.081	0.908	3.506
227.5	+760	−543	−2,100	−1,883	−1.883	−1.896	−0.971	53.241	0.0176	53.727	0.512	4.018
250	+760	−528	−2.003	−1.771	−1.771	−1.827	−0.778	52.463	0.0173	52.852	0.426	4.444
300	+760	−519	−1,787	−1,546	−1.546	−1.658	−1.605	50.858	0.0167	51.661	0.968	5.412
350	+760	−501	−1,570	−1.311	−1.311	−1.428	−1.424	49.434	0.0161	50.146	0.997	6.409
400	+760	−483	−1.353	−1.076	−1.076	−1.194	−1.223	48.211	0.0156	48.823	1.024	7.433
450	+760	−468	−1,136	−844	−0.844	−0.960	−1.006	47.205	0.0153	47.708	1.048	8.481
500	+760	−459	−920	−619	−0.619	−0.732	−0.782	46.423	0.0150	46.814	1.068	9.549
550	+760	−450	−704	−394	−0.394	−0.507	−0.519	45.874	0.0148	46.148	1.083	10.632
600	+760	−444	−487	−171	−0.171	−0.282	−0.309	45.565	0.0147	45.720	1.094	11.726
650	+760	−441	−271	+48	+0.048	−0.062	−0.068	45.497	0.0147	45.531	1.098	12.824
700	+760	−441	−54	+265	+0.265	+0.156	+0.171	45.668	0.0148	45.583	1.097	13.921
750	+760	−444	+162	+478	+0.478	+0.372	+0.405	46.073	0.0149	45.871	1.090	15.011
800	+760	−447	+379	+692	+0.692	+0.585	+0.631	46.704	0.0151	46.388	1.078	16.089
(b) FIRST TRIAL FOR ACCELERATION												
850	+760	−453	+595	+902	+0.902	+0.797	+0.846	47.550	0.0154	47.127	1.061	17.150
900	+651	−462	+812	+1.001	+1.001	+0.952	+0.991	48.541	0.0158	48.046	1.041	18.191
950	+543	−474	+1,028	+1,097	+1.097	+1.049	+1.069	49.610	0.0162	49.075	1.019	19.210
1,000	+434	−486	+1,245	+1,193	+1.193	+1.145	+1.140	50.750	0.0166	50.180	0.996	20.206
1,050	+326	−498	+1,461	+1,289	+1.289	+1.241	+1.209	51.959	0.0171	51.354	0.974	21.180
1,100	+217	−513	+1,678	+1,382	+1.382	+1.336	+1.271	53.230	0.0177	52.594	0.951	22.131
1,150	+109	−531	+1,894	+1,472	+1.472	+1.427	+1.324	54.554	0.0183	53.892	0.928	23.059
1,200	0	−549	+2,100	+1,551	+1.551	+1.512	+1.368	55.922	0.0190	55.238	0.905	23.964
1,250	0	−570	+2,100	+1,530	+1.530	+1.540	+1.360	57.282	0.0198	56.602	0.883	24.847
1,285	0	−594	+2,100	+1,506	+1.506	+1.518	+0.920	58.202	0.0204	57.742	0.606	25.453
1,300	+66	−612	+1.875	+1,329	+1.329	+1.418	+0.364	58.566	0.0206	58.384	0.257	25.710
1.350	+287	−618	+1,125	+794	+0.794	+1.062	+0.900	59.466	0.0213	59.016	0.847	26.557
1,400	+508	−639	+375	+244	+0.244	+0.519	+0.435	59.901	0.0217	59.683	0.838	27.395
1,425	+618	−651	0	−33	−0.033	+0.106	+0.044	59.945	0.0217	59.923	0.417	27.812
1.450	+618	−651	0	−33	−0.033	−0.033	−0.014	59.931	0.0217	59.938	0.417	28.229
1,500	+618	−651	0	−33	−0.033	−0.033	−0.028	59.903	.....	59.917	0.835	29.064

TABLE 1.—COMPUTATIONS FOR THE VELOCITY CURVE (*Continued*)

FEET	POUNDS				FEET PER SEC²		FEET PER SEC		LB PER LB	FT PER SEC	SECONDS	
s	P_e	R_t	R_g	P_n	a_s	a_a	Δv_s	v_s	f	V	Δt	t
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					(*c*) SECOND TRIAL FOR ACCELERATION							
800	+760	−447	+379	+692	+0.692	....	....	46.704	0.0151	....	....	16.089
850	+591	−453	+595	+733	+0.733	+0.712	+0.756	47.460	0.0154	47.082	1.062	17.151
900	+422	−462	+812	+772	+0.772	+0.752	+0.786	48.246	0.0157	47.853	1.045	18.196
950	+254	−471	+1,028	+811	+0.811	+0.792	+0.814	49.060	0.0160	48.653	1.028	19.224
1,000	+84	−480	+1,245	+849	+0.849	+0.830	+0.839	49.899	0.0163	49.480	1.011	20.235
1,025	0	−489	+1,353	+864	+0.864	+0.856	+0.427	50.326	0.0164	50.113	0.499	20.734
1,050	0	−492	+1,461	+969	+0.969	+0.916	+0.453	50.779	0.0166	50.553	0.495	21.229
1,100	0	−498	+1,678	+1,180	+1.180	+1.074	+1.047	51.826	0.0171	51.303	0.975	22.204
1,150	0	−513	+1,894	+1,381	+1.381	+1.280	+1.221	53.047	0.0176	52.437	0.954	23.158
1,200	0	−528	+2,100	+1,572	+1.572	+1.476	+1.374	54.421	0.0182	53.734	0.931	24.089
1,250	0	−546	+2,100	+1,554	+1.554	+1.563	+1.418	55.839	0.0190	55.130	0.907	24.996
1,285	0	−570	+2,100	+1,530	+1.530	+1.542	+0.958	56.797	0.0195	56.318	0.621	25.617
1,300	+66	−585	+1,875	+1,356	+1.356	+1.443	+0.380	57.177	0.0197	56.987	0.263	25.880
1,350	+287	−591	+1,125	+821	+0.821	+1.088	+0.943	58.120	0.0203	57.649	0.867	26.747
1.400	+508	−609	+375	+274	+0.274	+0.548	+0.470	58.590	0.0206	58.355	0.857	27.604
1,425	+618	−618	0	0	0	+0.137	+0.058	58.648	0.0206	58.619	0.426	28.030
1,450	+618	−618	0	0	0	0	0	58.648	0.0206	58.648	0.426	28.456
1,500	+618	−618	0	0	0	0	0	58.648	....	58.648	0.853	29.309

the interval from distance $s = 0$ to distance $s = 50$, is the gross vehicle weight (W_g) multiplied by the coefficient, or $30{,}000 \times 0.0206 = -618$ lb. Since the truck speed (V) is constant on the level approach grade, the tractive effort (P_e) at distance $s = 0$ is just sufficient to balance this resistance; that is, +618 lb. In the numerical work, the forces tending to decelerate the vehicle are marked negative and those tending to accelerate the vehicle are marked positive. The foregoing data are then recorded at $s = 0$ ft in Table 1.

The tractive resistance (R_t) used at distance $s = 50$ is −618 lb, as stated in the preceding paragraph. The engine tractive effort (P_e) at distance $s = 50$ is equal to $618 + (760 - 618) \times 50/140 = +669$ lb. The grade resistance (R_g) is equal to $30{,}000 \times 0.07 \times 50/140 = -750$ lb (see $s = 50$ ft under Cols. 1 to 4, Table 1). Taking the algebraic sum of the engine tractive effort, the tractive resistance, and the grade resistance, the net force P_n is found to be −699 lb; the acceleration (a_s) at the end of the interval between $s = 0$ and $s = 50$ is computed by dividing the net force by the total mass of the truck, $-699/1{,}000 = -0.699$ ft per sec per sec; and the average acceleration (a_a) during the passage of the truck from distance $s = 0$ to distance $s = 50$ is the arithmetic mean of the accelerations at the beginning and the end of the interval, $(0.000 - 0.699)/2 = -0.350$ ft per sec per sec (see $s = 50$ under Cols. 5, 6, and 7, Table 1).

The velocity change (Δv_s) in the interval from $s = 0$ to $s = 50$ is computed by multiplying the average acceleration by the time (Δt) taken for the truck to pass through the distance interval at an average velocity (V) equal to the

arithmetic mean of the velocities at the beginning and the end of the interval. It is necessary to assume an average velocity (V); then find the time for this velocity; compute the velocity change and the velocity at the end of the interval; and observe whether the mean velocity checks that assumed. Using an assumed average velocity of 58.500 ft per sec, the time is 50/58.5 = 0.855 sec, and the velocity change is $-0.350 \times 0.855 = -0.299$, giving a velocity of $58.667 - 0.299 = 58.368$ ft per sec at distance $s = 50$, and an average velocity in the interval of $(58.667 + 58.368)/2 = 58.518$ ft per sec. This velocity is used for the second trial, since it does not check the value previously assumed. The time for the second trial is 50/58.518 = 0.854 sec, and the velocity change is $-0.350 \times 0.854 = -0.299$ ft per sec. The velocity at the end of interval is $58.667 - 0.299 = 58.368$; and the mean velocity is the same as computed in the first trial (see $s = 50$, under Cols. 8, 9, 11, and 12, Table 1).

The time for the distance interval, 0.854 sec, is added to the previous value for total time. The coefficient of tractive resistance for the velocity at the end of the interval is read from the curve in Fig. 2, and found to be 0.0205 lb per lb (see $s = 50$, under Cols. 10 and 13, Table 1). The complete data for the second line of the table have thus been computed.

These several steps are repeated for the computation of the data for distance $s = 100$, and for the subsequent distance intervals. The computations for time and velocity may be performed very easily and quickly with a slide rule, with sufficient accuracy for the purpose. On certain types of rules, only one setting of the slide is required for each trial; the second trial is usually sufficiently close to the correct value, so that, in reality, the computations are much simpler than the description indicates.

In Fig. 3 the curves for the complete computations shown in Table 1 are plotted. The tractive-effort curve indicated by a broken line in Fig. 3(c) represents the first trial to increase the speed of the vehicle to the initial value of 40 miles per hr at the end of the descending grade. The corresponding velocity curve (Fig. 3(a)) is also indicated by a broken line. The tractive-effort curve shown by the solid line in Fig. 3(c) was projected as the second trial, and the corresponding velocity curve is also shown by a solid line in Fig. 3(a). The numerical values for the second trial are indicated in Table 1(c).

12. Time and Distance Loss

The time lost by the assumed vehicle for a given distance is the difference between the computed time for traversing this distance and the time required to cover the same distance at a constant speed equal to the initial speed, which is 40 miles per hr in the illustrative problem. Fig. 4 shows the time and distance lost for any point on the profile of the example. The computed time curve represents the total time, shown in Col. 13, Table 1, plotted against the distance for each 50-ft interval point on the profile, the plotted curve being that of the values for the second trial computation for points beyond distance $s = 800$ (see Table 1(c)). The time curve for uniform velocity is a straight line, with a slope of one second on the time scale to 58.667 ft on the distance scale.

The time loss at a given point on the profile, as compared with a uniform speed of 40 miles per hr, may be scaled from Fig. 4, as the difference between

the ordinates to the computed time curve and the time curve for uniform velocity, for the distance in question. For distance $s = 1{,}425$ the time loss scales 3.7 sec. Since this point is the end of the descending grade of the profile used in the example, the time loss at this point may be called the total for the profile.

The total time loss is the answer sought in most overpass grade problems, since it may be used as a basis for comparison with other grade lines between control points, and intermediate values for time loss are not important. For such cases, the total time loss can be calculated directly without plotting the time curves. In the example, the total computed time for the distance of 1,425 ft, in the second trial, is 28.03 sec. The time required to cover this distance at a constant speed of 40 miles per hr is $1{,}425/58.667 = 24.29$ sec. The total time loss is $28.03 - 24.29 = 3.74$ sec.

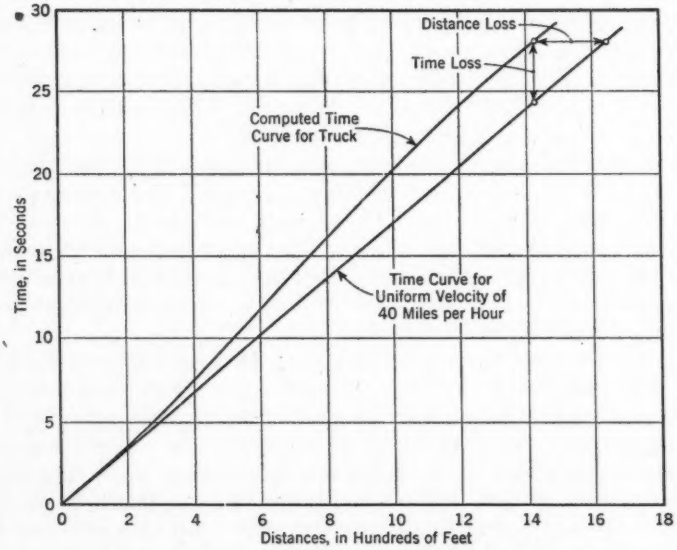

FIG. 4.—TIME-DISTANCE CURVES CORRESPONDING TO FIG. 3

The distance lost by the truck at a given point on the profile is defined as the product of the time loss in seconds at the given point and the governing initial velocity, using the unit of feet per second to obtain the distance loss in feet. In the example, the total distance loss for the profile, at $s = 1{,}425$, is $3.74 \times 58.667 = 219$ ft. In other words, when the truck in the example reaches $s = 1{,}425$, it is 219 ft back of the position it would have reached in the same time at a constant speed of 40 miles per hr. The distance loss may be scaled from Fig. 4, as the distance between the two time curves—that is, by measuring horizontally from a point on the computed time curve, corresponding to the actual distance or time in question, to the time curve for uniform velocity.

For determining time or distance, lost or gained, by scaling from a time-distance curve, the measurement should be made in all cases from the point in question on the computed curve to the curve for uniform velocity.

13. Use of Time Intervals, Rather than Distance Intervals, for Velocity-Curve Computations

It is practicable to use a computation procedure involving time intervals, instead of distance intervals, as used in the illustrative problem. This procedure would simplify the detailed computations for velocity slightly, since it would eliminate the necessity for trial determination of the time interval during which a certain average acceleration operates; but the use of this procedure would complicate the determination of the forces acting upon the vehicle at the end or the beginning of the constant time interval, and would make the plotting of the acceleration and velocity curves more difficult.

14. Graphic Methods for Determining Velocity Curves

Graphic methods for the construction of velocity curves, providing satisfactory accuracy for the type of problem under consideration, have been used in connection with similar studies in railway work, and there are apparently no circumstances that would prevent the adaptation of these graphic methods for studies of truck velocities. The description of such a method, including a bibliography, was presented in 1941 by A. I. Lipetz.[8] It is considered unlikely that the graphic method would prove advantageous, from the standpoint of saving time and labor, as compared with the numerical method described herein, for the short profiles under consideration in this study. In the comparison of different grade lines, such as might be encountered on different routes between control points in mountainous terrain, the use of a graphic method might be distinctly advantageous.

15. Effect of Height of Rise and Rate of Grade on Time Loss

In an effort to determine the factors that would provide the best designs for grade lines over grade separation structures on a freeway, computations of the velocity curves and the time loss for the truck described in Fig. 3 were made for a variety of grade lines. Values between 5 ft and 28 ft were used for the rise in grade. The velocities were not restored to exactly 40 miles per hr at the end of the descending grade in some cases, since it appeared that variations up to about 0.3 mile per hr would permit sufficiently accurate computation of time loss. An endeavor was made to use procedures for applying the tractive effort on the descending grade for the different profiles which followed the same general principles in all cases.

In Fig. 5 the time losses for the various profiles are plotted against the rise in grade. The curve drawn through points A to I is a quadratic parabola. From the relation between the plotted points and the parabola it is evident that the time loss is approximately proportional to the square of the rise in grade, and the four profiles studied for the 28-ft value of rise indicate that the time loss is not greatly affected by the rate of grade. These conclusions are based on the assumed truck-trailer combination described in Section 8, and on a definite general procedure for the application of the tractive effort of the truck;

[8] "Graphical Methods for Plotting Time-Speed-Distance Curves for Railway Trains," by A. I. Lipetz, *Transactions*, A.S.M.E., October, 1941, p. 603.

and they should not be applied directly for other truck designs or loads, or for grades that require a shifting of gears. However, it appears that the assumptions that time loss is approximately proportional to the square of the rise and is independent of the rate of grade may be used as a general basis for designing overpass grades, provided the same standard for vertical sight distance governs for the different profiles considered.

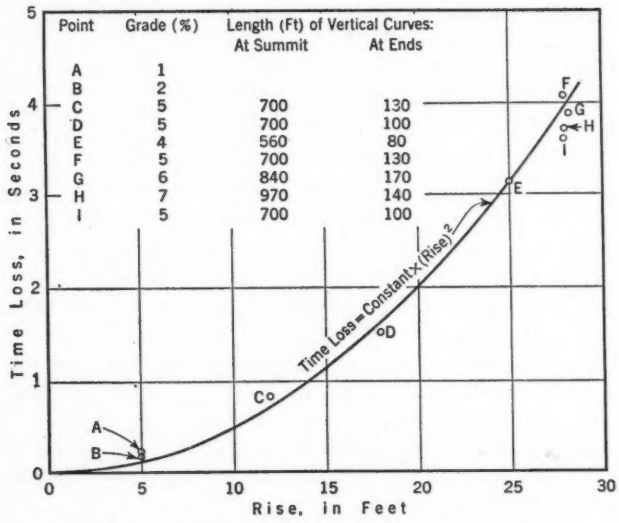

FIG. 5.—TIME LOSS FOR A TRUCK ON OVERPASS GRADES (90-HP MOTOR, HIGH GEAR; W_g = 30,000 LB; VERTICAL SIGHT DISTANCE, 500 FT)

16. EFFECT OF OVERPASS GRADES ON HIGHWAY CAPACITY

The time lost by a single truck on an up-and-down grade such as that discussed herein is a fairly definite quantity, subject to the aforementioned limitations on accuracy. The effect of a rise in grade to overpass a railroad or street on the carrying capacity of a single traffic lane or on the capacity of a complete highway facility is somewhat more difficult to evaluate. It is possible, however, to give some general comments relative to this question, which may be useful in the study of specific problems.

In 1942, O. K. Normann[9] presented a paper containing data and comments pertaining to the maximum theoretical capacity and the practical working capacity of various types of highways. In the remarks that follow, use is made of ideas expressed by Mr. Normann, with only a brief explanation of the fundamentals involved. Reference to the original source will provide the reader with a broader background for review of the remarks contained herein.

There is an average minimum spacing, which depends upon velocity, for vehicles moving in a single lane of a given highway, and this spacing may be determined by the proper test methods when traffic conditions on the highway

[9] "Results of Highway-Capacity Studies," by O. K. Normann, *Public Roads*, June, 1942, p. 57.

are suitable. On the basis of the average minimum spacing between vehicles for various speeds, it is possible to calculate the maximum theoretical traffic capacity of the traffic lane under consideration. Under actual operating conditions, the traffic volume will rarely reach the maximum theoretical capacity, since this capacity is based on continuously maintaining the minimum spacing. There is a definite practical working capacity for a given traffic lane of a given highway, however, and a working capacity for a given traffic facility, taken as a whole. The practical working capacity of a given section of highway is a relative value depending on various local conditions.

If a truck travels over a certain section of a highway which includes an overpass grade, and there is no other traffic on the section of highway at the time, the reduction in speed of the truck in moving over the grade rise affects the truck alone. That is, no other vehicle is delayed as a result of the reduction in speed of the truck. On the other hand, if the same section of highway is carrying its maximum theoretical traffic volume, for the assumed speed of 40 miles per hr (which condition may exist for a short time) and one of the vehicles moving over the section is a truck such as that of Section 8, it is evident that each vehicle following the truck over the grade rise will lose time in an amount equal to the time lost by the truck, provided the average minimum spacing is maintained through the section, since under the condition of maximum theoretical capacity there will be no passing. If a stream of traffic feeds into a section of highway containing an overpass grade at a speed of 40 miles per hr, at the average minimum spacing for this speed, and a considerable percentage of trucks is involved, the time lost by a truck at the grade rise will result in delay for each vehicle following the truck. The loss of time incurred by each additional truck will cause additional delay for the vehicles behind it, and congestion will eventually result at some point on the highway back of the grade rise. The maximum theoretical traffic capacity of the section of highway for a speed of 40 miles per hr will be reduced as a direct result of the presence of the grade rise, provided, of course, the trucks and cars do not regain the time lost at the overpass grade by operating at a speed in excess of 40 miles per hr at any place in the section.

The conditions outlined in the preceding paragraph are extreme. The practical working capacity of a lane of a divided multilane highway will probably be less than two thirds of its theoretical maximum capacity; and, although two, three, or more vehicles will be encountered in a group, spaced at or near the minimum distance, such groups will be separated by spacings greater than the minimum. Under these conditions, the reduction in speed of a given truck passing over a grade rise would result in delay for the vehicles following the truck and not separated from it or the preceding vehicle by a spacing sufficiently long to obviate the effects of speed reduction by the vehicles ahead.

This last point may be clarified by an example, in which it is assumed that, on one lane of the section of road approaching the grade rise, vehicles are traveling in groups of five, at 100-ft average minimum spacing for the initial speed of 40 miles per hr; and these groups of five vehicles are spaced 1,000 ft apart, net. Then, if a truck included in one of the groups incurs a time loss and corresponding distance loss on the given profile, only the vehicles behind

the truck in its group will be delayed as a result of the truck time loss, unless the distance loss for the last vehicle in this group amounts to more than 900 ft; that is, the original group spacing of 1,000 ft, less the average minimum spacing between vehicles for the speed involved. If the latter condition occurs, vehicles in the group following that containing the truck will incur delay; if the delay to this second group is sufficiently great, a third group also will be delayed, and so on. A succession of grade rises in the profile under consideration would involve cumulative time losses. From these considerations, it appears that in a practical problem the number of vehicles that would be delayed by a reduction in speed of a truck on a given profile can be determined only on the basis of mathematical probability, taking proper account of the average minimum spacing between vehicles, the average number of vehicles in a group, the average spacing between groups, and the proportion of vehicles that are unable to maintain speed on the given profile.

Furthermore, under working conditions, vehicles following the truck would not necessarily be prohibited from passing the truck while its velocity is less than 40 miles per hr, or from increasing speed to a value above 40 miles per hr and passing the truck after traveling over the grade rise; so it is possible that, on a sufficiently long section of highway involving one overpass grade, some of the vehicles in a group following a truck at the grade rise might pass the truck and thus permit the remainder of the vehicles to make up their lost time, with the result that none of the vehicles except the truck would incur a time loss for the section of highway under consideration. Finally, it is possible that the truck might operate at a speed greater than 40 miles per hr after passing the overpass grade and thus overcome the loss of time on the grade.

The foregoing remarks indicate the wide range of conditions that may be encountered in a study of the effect of grade rises on the capacity of a highway. Ordinary conditions of the profile and ordinary proportions of truck traffic apparently are not likely to justify the construction of an extra traffic lane on overpass grades to permit passing of heavily loaded vehicles. However, such extra lanes are often justified and have been built at many locations on rural highways where fairly long and steep grades are involved, to eliminate excessive delays to traffic.

17. Time Loss on Rising Grade Without Compensating Falling Grade

As may be seen from a study of Fig. 3, the force due to grade is large in comparison with the tractive effort available from the engine for heavily loaded vehicles on ordinary overpass grades. In cases where the grade rise is not accompanied by a corresponding drop in grade beyond the overpass, a considerable increase in time loss prior to a restoration of the velocity to its initial value is to be expected, as compared to time loss in the case where the rise is compensated by an equal fall. Fig. 6 shows complete force, acceleration, velocity, and time curves for two profiles involving a rise of 12 ft. The broken lines represent values for a profile with a 12-ft drop in grade beyond the overpass, and the solid lines represent values for a profile with a level grade beyond the overpass. The total time loss for the first case is 0.83 sec and the distance loss

49 ft. For the second case the time and distance losses are 1.50 sec and 88 ft, or about 80% greater than for the first case.

The rise of 12 ft involved in these cases is an ordinary value for urban grade separations between streets, and the value of 1.5 sec for time loss is small. For a rise of 25 ft or more, not compensated by a drop in grade, it appears that excessive time loss might result. To permit the use of a convenient scale, the time curves between the beginning of the profile and distance $s = 500$ are not shown in Fig. 6.

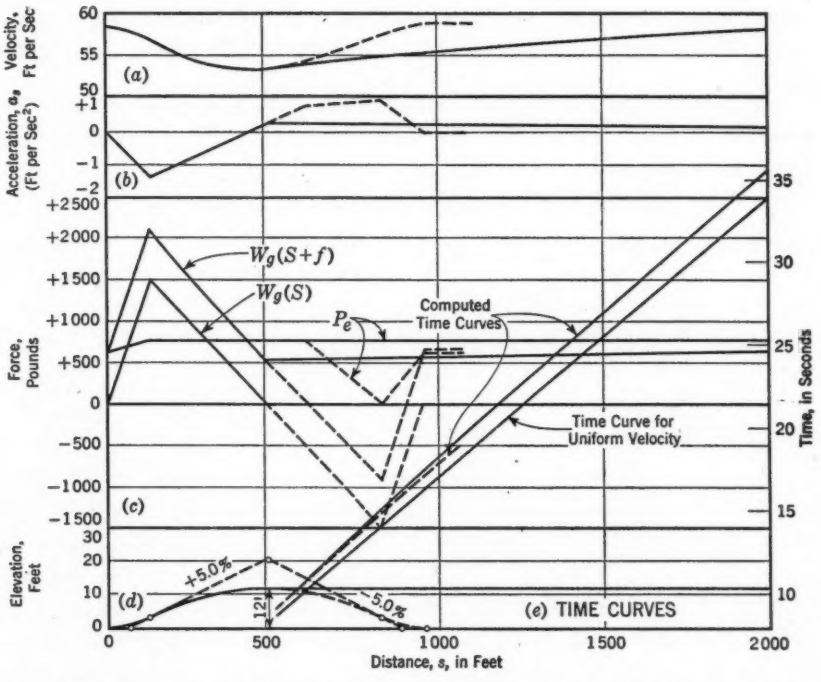

FIG. 6.—TRUCK OPERATION CURVES FOR A GRADE RISE WITH OR WITHOUT A COMPENSATING FALL

18. CONCLUSION

This paper is presented with the primary purpose of calling attention to the method of obtaining the velocity curve for a given vehicle traveling over a given profile. Although the original studies on which the paper is based concern overpass grades, and the examples are taken from those studies, the applicability of the outlined methods to a wide variety of grade problems is evident.

It is felt that the comments regarding the effect of low rises in grade, on truck speed and highway traffic capacity, may be useful in a field wider than that of the overpass grades for which they were originally formulated, and that they are of sufficient interest to warrant their inclusion with the exposition of the method for velocity-curve computation.

It is hoped that the method and corollary ideas will provide the engineer with improved means for selecting the best grade line, from the standpoint of operation of trucks and mixed highway traffic, not only for short adverse grades, but for longer profiles as well.

19. Acknolwedgment

The methods outlined were developed under the direction of H. G. Bossy, of the Texas Highway Department. For his numerous suggestions during the work, and for his aid in preparing the paper, Mr. Bossy deserves special credit. The work was done while the writer was employed as designing engineer by the Texas Highway Department of which D. C. Greer, M. ASCE, is state highway engineer, and J. L. Dickson was engineer of road design.

The writer has also received assistance, in the final preparation of the manuscript, from the Civil Engineering Department of the University of Texas at Austin.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

SPACE RESECTION PROBLEMS IN PHOTOGRAMMETRY

By P. H. Underwood,[1] M. ASCE

Synopsis

The paper gives a solution of the resection problem in space to find the position of a camera station and to obtain orientation data for a vertical photograph with a small amount of tilt by applying methods of mathematical calculation combined with some graphical work.

In solutions of this kind, approximate methods must be used for some of the steps. In this paper such approximations play an important part. The solution is based upon the use of two three-sided pyramids with a common vertex at the camera station, with sides in common planes, and with triangular bases—the one in the plane of the photograph and the other in the plane of the control points on the ground. The vertex angles of these pyramids are calculated by methods not very dissimilar to those commonly used. Then the lengths of the side edges of the ground pyramid are calculated, by use of the cosines of the base angles to expedite the work.

In solving next for the coordinates of the camera station, squared terms are introduced to make it easy to reach a precise solution by approximations. A method of "position circles" to obtain the elements of exterior orientation is developed and it is believed that this considerably facilitates this part of the solution. A method is also given for finding the tilt, swing, and azimuth of the principal line by mathematical calculations. A numerical example is added to illustrate the theoretical methods.

Theoretical Analysis

The determination of the elements of exterior orientation of a so-called vertical photograph is a problem that has received much consideration and has been treated in various ways. A useful list of references has been published

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by February 1, 1947.

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by R. Hugershoff and H. Cranz.[2] Earl Church,[3] Assoc. M. ASCE, has also done considerable work on this subject in the United States. The writer proposes to outline a method, certain steps of which he believes to have advantages over other methods in current use.

The letter symbols are defined where they first appear, in text or by illustration, and are assembled alphabetically, for convenience of reference, in the Appendix.

The problem may be outlined thus: A vertical photograph of the ordinary type with a certain small angle of tilt is being analyzed. On this photograph are three points of control which are suitably located for determining the elements of exterior orientation; that is, points not near the center of the photograph and not close to one another. The rectangular coordinates of these three points with respect to axes on the photograph are known. The y-axis is vertical and the x-axis horizontal, the origin being at the principal point of the picture. Let these three points be designated a, b, and c (Fig. 1) with coordinates x_a, y_a, x_b, y_b, x_c, and y_c, respectively. Let the corresponding points on the ground be A, B, and C, whose known coordinates with respect to a coordinate system on the ground are X_A, Y_A, Z_A, etc. The focal length of the camera f has supposedly been determined by calibration and is known. The problem is to find the swing s or angle made by the principal line with the y-axis on the photograph, the coordinates of the camera station on the ground control system, and the azimuth of the principal line on the ground-control system.

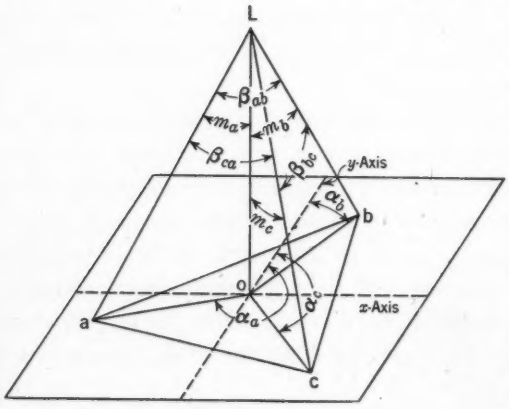

FIG. 1

The first step is to compute the azimuths on the plate α_a, α_b, and α_c, reckoned clockwise from the positive end of the y-axis, of the lines oa, ob, and oc (o being the principal point), the lengths of these lines, the lengths of the sides of the triangle abc, the lengths of the edges of the plate pyramid, La, Lb, and Lc (L, the vertex of the pyramid, is the perspective center, or lens center), and the angles aLo, bLo, and cLo which will be designated m_a, m_b, and m_c, respectively. Thus:

$$\tan \alpha_a = \frac{x_a}{y_a} \quad \ldots\ldots (1a)$$

$$\tan \alpha_b = \frac{x_b}{y_b} \quad \ldots\ldots (1b)$$

[2] "Grundlagen der Photogrammetrie aus Luftfahrzeugen," by R. Hugershoff and H. Cranz, Konrad Wittwer, Stuttgart, 1919.

[3] "Manual of Photogrammetry," Am. Soc. of Photogrammetry, Chapter XII, 1944.

and

$$\tan \alpha_c = \frac{x_c}{y_c} \quad \ldots\ldots (1c)$$

Also,

$$\overline{oa} = \sqrt{x^2_a + y^2_a} \quad \ldots\ldots (2a)$$

$$\overline{ob} = \sqrt{x^2_b + y^2_b} \quad \ldots\ldots (2b)$$

and

$$\overline{oc} = \sqrt{x^2_c + y^2_c} \quad \ldots\ldots (2c)$$

As a check, $\overline{oa} = \dfrac{x_a}{\sin \alpha_a} = \dfrac{y_a}{\cos \alpha_a}$, etc. Furthermore,

$$\overline{ab} = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} \quad \ldots\ldots (3a)$$

$$\overline{bc} = \sqrt{(x_c - x_b)^2 + (y_c - y_b)^2} \quad \ldots\ldots (3b)$$

and

$$\overline{ac} = \sqrt{(x_a - x_c)^2 + (y_a - y_c)^2} \quad \ldots\ldots (3c)$$

Finally,

$$\overline{La} = \sqrt{\overline{ao}^2 + f^2} \quad \ldots\ldots (4a)$$

$$\overline{Lb} = \sqrt{\overline{bo}^2 + f^2} \quad \ldots\ldots (4b)$$

and

$$\overline{Lc} = \sqrt{\overline{co}^2 + f^2} \quad \ldots\ldots (4c)$$

The next step is to solve for the angles aLb, bLc, and cLa at the vertex of the pyramid, calling these angles for brevity β_{ab}, β_{bc}, and β_{ca}, respectively. These calculations are somewhat similar to those described by Professor Church.[4] Solve for these angles by applying the cosine formula for a plane triangle in succession to the triangles aLb, bLc, and cLa. Thus,

$$\cos \beta_{ab} = \frac{-\overline{ab}^2 + \overline{La}^2 + \overline{Lb}^2}{2\,\overline{La} \times \overline{Lb}} \quad \ldots\ldots (5a)$$

$$\cos \beta_{bc} = \frac{-\overline{bc}^2 + \overline{Lb}^2 + \overline{Lc}^2}{2\,\overline{Lb} \times \overline{Lc}} \quad \ldots\ldots (5b)$$

and

$$\cos \beta_{ca} = \frac{-\overline{ac}^2 + \overline{Lc}^2 + \overline{La}^2}{2\,\overline{Lc} \times \overline{La}} \quad \ldots\ldots (5c)$$

Also solve for m_a, m_b, and m_c by the formulas:

$$\tan m_a = \frac{\overline{ao}}{f} \quad \ldots\ldots (6a)$$

$$\tan m_b = \frac{\overline{bo}}{f} \quad \ldots\ldots (6b)$$

and

$$\tan m_c = \frac{\overline{co}}{f} \quad \ldots\ldots (6c)$$

[4] "Manual of Photogrammetry," Am. Soc. of Photogrammetry, Chapter XII, 1944, p. 537.

These calculations may be checked by the formulas:

$$\sin m_a = \frac{\overline{ao}}{\overline{La}}, \qquad \sin m_b = \frac{\overline{bo}}{\overline{Lb}}, \qquad \text{and} \qquad \sin m_c = \frac{\overline{co}}{\overline{Lc}};$$

or by

$$\cos m_a = \frac{f}{\overline{La}}, \qquad \cos m_b = \frac{f}{\overline{Lb}}, \qquad \text{and} \qquad \cos m_c = \frac{f}{\overline{Lc}}.$$

The various steps in the solution that is to follow necessitate finding the coordinates with respect to the ground-survey system of the camera station L. After finding the coordinates of L the remainder of the solution includes finding the tilt, the swing, and the azimuth with respect to the ground-survey system of the principal plane of the photograph. It is now proposed to outline the various steps required to determine the coordinates of point L.

When the camera station L is connected to the three control points on the ground by the lines LA, LB, and LC a triangular pyramid may be considered to be formed with the vertex at point L and with the triangle ABC forming its base. The base of this pyramid will generally be inclined to the horizontal since the three control points are likely to be at different elevations.

The inclined lengths of the sides of the base may be calculated by the formulas:

$$\overline{AB} = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (Z_B - Z_A)^2} \quad \ldots\ldots (7a)$$

$$\overline{BC} = \sqrt{(X_C - X_B)^2 + (Y_C - Y_B)^2 + (Z_C - Z_B)^2} \quad \ldots\ldots (7b)$$

and

$$\overline{CA} = \sqrt{(X_A - X_C)^2 + (Y_A - Y_C)^2 + (Z_A - Z_C)^2} \quad \ldots\ldots (7c)$$

For later use the horizontal lengths of these sides should also be computed by dropping the Z-terms from Eqs. 7. If the Z-differences are small, the calculations may be checked by the approximate formula,

$$\text{(Slope length)} - \text{(horizontal length)} = \frac{(Z\text{-difference})^2}{2\ \text{(horizontal, or slope, length)}} \quad \ldots (8)$$

A graphical solution to find approximate lengths of the lines LA, LB, and LC should now be made. To do this the angles β (Fig. 1) are laid off with a

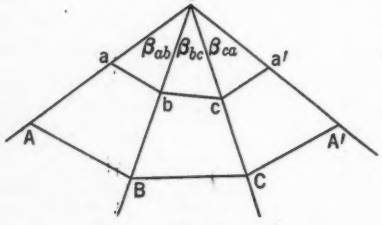

Fig. 2

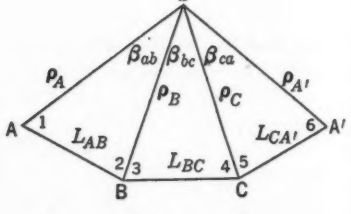

Fig. 3

protractor, or by the method of tangents, to give a development of the sides of the plate and ground pyramids, which have a common vertex and common side faces (see Fig. 2). The construction of the angles may be checked by

scaling off on these lines the distances La, Lb, and Lc, respectively, and then scaling from the drawing the lengths of ab, bc, and ca, and comparing with the previously calculated values.

To draw the base of the developed ground pyramid and thus determine the lengths of LA, LB, and LC the computer makes use of two conditions: First, that the lengths of the sides of this pyramid included between the lines La, Lb, Lc, and La′ must be equal in order to the corresponding inclined lengths AB, BC, and CA and, second, that the length of LA must be equal to that of LA′ since, of course, each represents the same edge of the ground pyramid. Graphically this problem cannot be solved directly but must be done by a trial-and-error method. After drawing AB to scale, parallel or approximately parallel to ab, strike an arc with radius equal to BC and center at point B to cut Lc at point C, and another arc with radius of length to scale equal to line CA, to cut La′ at point A′. If LA′ is not equal to LA (and it probably will not be) or nearly so, another trial may be made, starting at a point near A which is judged to meet the conditions more nearly. (A graphical method of finding the new position of point A, which, however, involves considerable labor, will be outlined subsequently.) In this way approximate lengths of LA, LB, and LC may be obtained, the precision of which will depend upon the scale of the drawing, the care with which the drafting is done, and the degree to which the approximation is carried. As a matter of fact the analytical solution which is to follow, although facilitated by having closely approximate values of LA, LB, and LC at the start, may be made from no more accurate initial values than can be found from the approximate relations $\frac{\overline{LA}}{\overline{La}} = \frac{\overline{AB}}{\overline{ab}} = \frac{\overline{CA}}{\overline{ca}}$, to find the length LA, averaging to obtain a better result. The degree of approximation in the value of LA thus obtained is dependent, of course, upon the tilt of the picture and the differences in elevation between control points.

In Fig. 3 the radial lines are designated ρ and the angles at the base of the pyramid are numbered for simplicity. Beginning with the approximate value of LA found by one of the methods just outlined, ρ_A, ρ_B, ρ_C, and $\rho_{A'}$ are computed in turn from the known quantities, the slope lengths AB, BC, and CA or L_{AB}, L_{BC}, and $L_{CA'}$, and the known angles β at the vertex of the pyramid. Since it is desired also to determine values of the angles 1, 2, 3, 4, 5, and 6, Fig. 3, these calculations will be made by use of the sine law. Thus,

$$\sin \angle (2) = \frac{\rho_A \sin \beta_{ab}}{L_{AB}} \qquad (9a)$$

$$\angle (1) = 180° - [\beta_{ab} + \angle (2)] \qquad (9b)$$

and

$$\rho_B = \frac{\rho_A \sin \angle (1)}{\sin \angle (2)} = \frac{L_{AB} \sin \angle (1)}{\sin \beta_{ab}} \qquad (9c)$$

The sides ρ_C and $\rho_{A'}$ are then computed in turn, first solving for ρ_C from ρ_B and then for $\rho_{A'}$ from ρ_C by similar formulas.

Of course, the value of $\rho_{A'}$ should be equal to ρ_A but it almost certainly will not prove that way since the value of ρ_A is only approximate. This first solu-

tion may be considered a trial one from which the engineer may derive, by the method now to be developed, a more accurate one. If this second solution does not give a sufficiently close check between $\rho_{A'}$ and ρ_A, further trials, as needed, may be made to yield a final solution with any required degree of precision.

A relationship between small changes in ρ_A and corresponding changes in $\rho_{A'}$ may be established by the method of rates and differentials. In the triangle LAB, applying the cosine formula:

$$L^2_{AB} = \rho^2_A + \rho^2_B - 2\,\rho_A\,\rho_B \cos \beta_{ab} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

In Fig. 3, L_{AB} and β_{ab} may be considered constant and ρ_A and ρ_B, variables. Then, $0 = 2\,\rho_A\,d\rho_A + 2\,\rho_B\,d\rho_B - 2\,\rho_A \cos \beta_{ab}\,d\rho_B - 2\,\rho_B \cos \beta_{ab}\,d\rho_A$; whence $\rho_B\,d\rho_B - \rho_A \cos \beta_{ab}\,d\rho_B = -\rho_A\,d\rho_A + \rho_B \cos \beta_{ab}\,d\rho_A$. Consequently,

$$\frac{d\rho_B}{d\rho_A} = -\frac{\rho_A - \rho_B \cos \beta_{ab}}{\rho_B - \rho_A \cos \beta_{ab}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11a)$$

Similarly, it can be shown that

$$\frac{d\rho_C}{d\rho_B} = -\frac{\rho_B - \rho_C \cos \beta_{bc}}{\rho_C - \rho_B \cos \beta_{bc}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11b)$$

and

$$\frac{d\rho_{A'}}{d\rho_C} = -\frac{\rho_C - \rho_{A'} \cos \beta_{ca}}{\rho_{A'} - \rho_C \cos \beta_{ca}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11c)$$

For simplicity, let the product of Eqs. 11 be

$$\frac{d\rho_B}{d\rho_A} \times \frac{d\rho_C}{d\rho_B} \times \frac{d\rho_{A'}}{d\rho_C} = \frac{d\rho_{A'}}{d\rho_A} = u \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The constant u may be found from Eqs. 11 directly or it may be found more simply from a relationship between functions of the known angles 1, 2, 3, 4, 5, and 6, Fig. 3. From the triangles formed by dropping perpendiculars from points A, B, C, and A′ upon the adjacent lines from L—that is, upon LA, LB, LC, and LA′, Fig. 3—the numerators in Eqs. 11 are found to be: $L_{AB} \cos \angle$ (1), $L_{BC} \cos \angle$ (3) and $L_{CA'} \cos \angle$ (5), respectively, whereas the corresponding denominators are $L_{AB} \cos \angle$ (2), $L_{BC} \cos \angle$ (4), and $L_{CA'} \cos \angle$ (6). Substituting these values in Eq. 12, the lengths L cancel out and

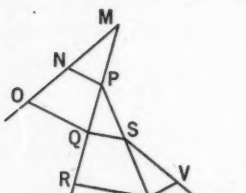

FIG. 4

$$u = \frac{d\rho_{A'}}{d\rho_A} = -\frac{\cos \angle\ (1)}{\cos \angle\ (2)} \times \frac{\cos \angle\ (3)}{\cos \angle\ (4)} \times \frac{\cos \angle\ (5)}{\cos \angle\ (6)} \quad \ldots\ldots\ldots\ldots (13)$$

This value of u may be found graphically as follows: After making a trial graphical solution as already described (to make La′ = La in Fig. 2) draw a line MO, Fig. 4, and on this line lay off to any convenient scale MN equal in length to $L_{AB} \cos \angle$ (1) and NO equal to $L_{AB} \cos \angle$ (2) and on an intersecting

line MR lay off MP equal to $L_{BC} \cos \angle$ (4). Connect points N and P and draw line OQ parallel to line NP. Then by the well-known formula for proportionality of intercepts between parallel lines, $\overline{MN}/\overline{NO} = \overline{MP}/\overline{PQ}$. Now lay off QR equal to $L_{BC} \cos \angle$ (3), draw an intersecting line PU through point P, lay off PS equal to $L_{CA'} \cos \angle$ (5) and draw RT parallel to QS. Then $\overline{PS}/\overline{St} = \overline{PQ}/\overline{QR}$. By a similar construction lay off TU equal to $L_{CA'} \cos \angle$ (6), draw an intersecting line SW through point S and draw the parallel lines TV and UW. Then $\overline{ST}/\overline{TU} = \overline{SV}/\overline{VW}$.

By multiplying together the three sets of ratios just obtained, PQ and ST cancel out and $\dfrac{\overline{MN} \times \overline{PS}}{\overline{NO} \times \overline{TU}} = \dfrac{\overline{MP} \times \overline{SV}}{\overline{QR} \times \overline{VW}}$ or, rearranged,

$$\frac{\overline{SV}}{\overline{VW}} = \frac{\overline{MN} \times \overline{QR} \times \overline{PS}}{\overline{NO} \times \overline{MP} \times \overline{TU}} \qquad (14)$$

By construction, $\overline{MN} = L_{AB} \cos \angle$ (1), $\overline{QR} = L_{BC} \cos \angle$ (3), $\overline{PS} = L_{CA'} \cos \angle$ (5); $\overline{NO} = L_{AB} \cos \angle$ (2), $\overline{MP} = L_{BC} \cos \angle$ (4), and $\overline{TU} = L_{CA'} \cos \angle$ (6). Substituting these values in Eq. 14*a*, the ratio $\dfrac{\overline{SV}}{\overline{VW}}$ is found equal to u as expressed in Eq. 13. In other words, by dividing the scaled length of line SV by that of line VW in Fig. 4, the value of the ratio u is obtainable.

For infinitesimal changes in ρ_A and $\rho_{A'}$, $d\rho_{A'} = u\, d\rho_A$ and, for small changes of finite amount,

$$\Delta\rho_{A'} = u\, \Delta\rho_A \qquad (15a)$$

—approximately. To effect a solution of the given problem the two edges of the ground pyramid must be made equal—that is, in Fig. 3, LA′ must be made equal to LA. Thus,

$$\rho_A + \Delta\rho_A = \rho_{A'} + \Delta\rho_{A'} \qquad (15b)$$

In Eq. 15*b*, substituting for $\Delta\rho_{A'}$ its value from Eq. 15*a* and solving for $\Delta\rho_A$:

$$\Delta\rho_A = \frac{\rho_{A'} - \rho_A}{1 - u} \qquad (16)$$

It should be noted that usually when the control points have been properly chosen the angles, 1, 2, 3, 4, 5, and 6, will all be acute angles with positive cosines and thus u will be a negative quantity, and $\Delta\rho_A$ and $\Delta\rho_{A'}$ will be of opposite sign.

For use in graphical solutions (and, in fact, for analytical solutions except when a highly accurate solution is desired) $\Delta\rho_A$ and $\Delta\rho_{A'}$ may be found graphically in Fig. 4 by laying off SW equal in length to $\rho_{A'} - \rho_A$ before drawing the parallel lines UW and TV. Then line SV will be equal, to scale, to $\Delta\rho_{A'}$ and VW to $\Delta\rho_A$. The signs may be noted separately.

After finding the corrections to ρ_A and $\rho_{A'}$ as indicated, a corrected value of ρ_A may be found and a second solution for ρ_B, ρ_C, and $\rho_{A'}$ carried through in the same way as the first—that is, either graphically or analytically—to reach more accurate results than in the first. Unless a very precise solution is required, a third approximate solution will usually not be necessary. Any sub-

sequent solution after the first may be checked approximately as ρ_B and ρ_C are found, because the corrections to the old values of these quantities to obtain the new ones may be found, from the value of $\Delta\rho_A$ used, by the relationships:

$$\Delta\rho_B = -\frac{\cos \angle (1)}{\cos \angle (2)} \Delta\rho_A \ldots\ldots\ldots\ldots\ldots\ldots (17a)$$

and

$$\Delta\rho_C = -\frac{\cos \angle (3)}{\cos \angle (4)} \Delta\rho_B \ldots\ldots\ldots\ldots\ldots\ldots (17b)$$

After two approximate solutions have been made a third one may be made, to give highly precise results, by using the values of u obtained in the first two solutions to compute an average value of this as ρ_A changes from its value obtained in the second solution to a predicted value to be obtained by a third solution. Thus u will change a certain amount as ρ_A changes from its first value to its second. Using the value of u from the second solution the value of ρ_A is calculated for a third solution. Assuming that, as ρ_A changes, u changes in proportion, a value of u may be calculated for the proposed third solution. Then the change in ρ_A may be recalculated by using a value of u which is an average of the value after the second solution and the value obtained in the manner mentioned for a third solution. Unless u is changing rapidly this value should lead to a value of ρ_A which is close to the true value.

Favorable and Unfavorable Positions of the Three Control Points.—As previously stated, control points should be well separated on the photograph, preferably located so as to form a nearly equilateral triangle when adjoining pairs are connected, with the principal point near the center of the triangle. Under these circumstances the angles 1 to 6, Fig. 3, will be acute angles whose cosines are of considerable size and not changing very rapidly. Furthermore, the value of u will be nearly unity and will not change rapidly as ρ_A is modified from one approximate solution to the next. If one of the angles 1 to 6 should be a right angle or nearly so, the solution would be much more unstable, because the cosine would be zero, or very small, and would be changing at a rapid rate with any modification of sides in effecting a solution. One side might be changed by a considerable amount without changing an adjoining one very appreciably. Thus small errors in the coordinates of the control points, or small errors in the solution, might result in large errors in the calculated lengths of the edges of the ground pyramid.

Coordinates of Station L.—Having determined the values of LA, LB, and LC, the next step in the solution is to obtain the coordinates, in space, of the camera station L. If X_L, Y_L, and Z_L represent these coordinates and if the formula for finding the distance between them is applied (see Eqs. 7):

$$(X_L - X_A)^2 + (Y_L - Y_A)^2 + (Z_L - Z_A)^2 - \overline{LA}^2 = 0 \ldots\ldots (18a)$$

$$(X_L - X_B)^2 + (Y_L - Y_B)^2 + (Z_L - Z_B)^2 - \overline{LB}^2 = 0 \ldots\ldots (18b)$$

and

$$(X_L - X_C)^2 + (Y_L - Y_C)^2 + (Z_L - Z_C)^2 - \overline{LC}^2 = 0 \ldots\ldots (18c)$$

Since it is impractical to solve Eqs. 18 directly for the unknown X_L, Y_L, and Z_L, approximate methods must be used again. A graphical or analytical solution of the three-point problem may be used to obtain approximate values of X_L and Y_L. This solution utilizes the angles about the principal point on the picture as approximations of the corresponding angles in the map plane about the vertical point, or horizontal projection of the camera station. In the graphical method the three control points are plotted on a map by the use of their coordinates; after taking off the three lines oa, ob, and oc, from the photograph, on a piece of transparent paper, cloth, or celluloid, the transfer is slipped around on the map until the three lines pass through the corresponding mapped points. The principal point is then pricked through to give the approximate location, in plan, of the camera station from which to scale off the approximate values of X_L and Y_L. Those who have a calculating machine available and are familiar with the method of solving for the coordinates of a point by the three-point problem, working directly from the coordinates of the control points, very likely can make an analytical solution more quickly than they can the graphical one, unless the points have already been mapped. No very exact solution need be made (nor can it be made) in either case, since the horizontal angles are only approximately known.

A sufficiently accurate value of Z_L may be known from the height of flight or from that of an adjoining photograph in the same flight. If not thus known it may be calculated to a sufficient degree of accuracy in the following way by comparing distances on the map with corresponding ones on the photograph. The distances OA, OB, and OC from the camera station O, to each of the control points, in turn, are scaled on the map. Then, approximately,

$$Z_L - Z_A = \frac{\overline{OA} \times f}{\overline{oa}} \quad \ldots\ldots (19a)$$

$$Z_L - Z_B = \frac{\overline{OB} \times f}{\overline{ob}} \quad \ldots\ldots (19b)$$

and

$$Z_L - Z_C = \frac{\overline{OC} \times f}{\overline{oc}} \quad \ldots\ldots (19c)$$

Since the method is approximate there will be discrepancies in the values of Z_L obtained from Eqs. 19. An average may be taken.

Next, let $X_L = X_{LO} + \Delta X$, $Y_L = Y_{LO} + \Delta Y$, and $Z_L = Z_{LO} + \Delta Z$, in which X_{LO}, Y_{LO}, and Z_{LO} are the approximate values of the coordinates of the camera station just found and ΔX, ΔY, and ΔZ are corrections to these to obtain the true values X_L, Y_L, and Z_L. By substituting these values of X_L, Y_L, and Z_L in the three simultaneous equations already written and combining and simplifying numerical values three simultaneous equations in ΔX, ΔY, ΔZ are obtained, as follows:

$$a_1 \Delta X + b_1 \Delta Y + c_1 \Delta Z + \tfrac{1}{2}(\Delta X^2 + \Delta Y^2 + \Delta Z^2) + q_1 = 0 \ldots (20a)$$

$$a_2 \Delta X + b_2 \Delta Y + c_2 \Delta Z + \tfrac{1}{2}(\Delta X^2 + \Delta Y^2 + \Delta Z^2) + q_2 = 0 \ldots (20b)$$

and

$$a_3 \Delta X + b_3 \Delta Y + c_3 \Delta Z + \tfrac{1}{2} (\Delta X^2 + \Delta Y^2 + \Delta Z^2) + q_3 = 0 \ldots . (20c)$$

In Eqs. 20,

$$a_1 = X_{LO} - X_A \ldots . (21a)$$

$$b_1 = Y_{LO} - Y_A \ldots . (21b)$$

$$c_1 = Z_{LO} - Z_A \ldots . (21c)$$

and

$$q_1 = \tfrac{1}{2} (a^2_1 + b^2_1 + z^2_1 - \overline{\mathrm{LA}}^2) \ldots . (21d)$$

Corresponding terms in Eqs. 20 are written similarly. If good approximations have been made to the true values of the coordinates of the camera station, ΔX, ΔY, and ΔZ will be relatively small quantities and the further solution will be simplified. However, by the approximate method about to be explained a solution for ΔX, ΔY, and ΔZ, accurate to a high degree of precision, may be made without excessive labor even though the approximation in coordinates is not close. To simplify Eqs. 20 assume that $\frac{1}{2}(\Delta X^2 + \Delta Y^2 + \Delta Z^2) = k$ and write

$$a_1 \Delta X + b_1 \Delta Y + c_1 \Delta Z + k + q_1 = 0 \ldots . (22a)$$

$$a_2 \Delta X + b_2 \Delta Y + c_2 \Delta Z + k + q_2 = 0 \ldots . (22b)$$

and

$$a_3 \Delta X + b_3 \Delta Y + c_3 \Delta Z + k + q_3 = 0 \ldots . (22c)$$

Eqs. 22 are now to be solved for ΔX, ΔY, and ΔZ. The first step is to eliminate ΔZ between Eq. 22*a* and Eq. 22*b* and then between Eq. 22*a* and Eq. 22*c*, each time finding coefficients for the k-terms. This will result in two simultaneous equations in ΔX and ΔY in which there will be k-terms. Usually the coefficients of K will be small because the c-coefficients are likely to be nearly of the same magnitude, since Z_L is large compared with the differences in elevation between the control points. Next eliminate ΔY between the equations just found, again carrying through the coefficients of k. Neglecting k, solve for ΔX and then for ΔY and ΔZ by substituting back in the preceding equations. With these values of ΔX, ΔY, and ΔZ compute a value of k and then solve again, correcting this time for the value of k. This will result in a closer approximation to the values of ΔX, ΔY, and ΔZ. If considered necessary a new value of k can be found and the substitution made for a third solution, and similarly for other solutions, to approach values as precise as necessary. It will be found, however, that a rapid approach to true values is made by this method. The addition of these values to X_{LO}, Y_{LO}, and Z_{LO}, respectively, gives X_L, Y_L, and Z_L, which may be checked if desired by substituting back into Eqs. 18.

Let the angles between the vertical LV and each of the lines, LA, LB, and LC, in turn be designated M_A, M_B, and M_C, respectively. (Point V is the vertical intersection in the XY-plane). Then

$$\cos M_A = \frac{Z_L - Z_A}{\overline{\mathrm{LA}}} \ldots . (23a)$$

$$\cos M_B = \frac{Z_L - Z_B}{\overline{\mathrm{LB}}} \ldots . (23b)$$

and

$$\cos M_C = \frac{Z_L - Z_C}{\overline{LC}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23c)$$

It will be necessary later to know the azimuths in the ground-survey system of the lines VA, VB, and VC, which may as well be found at this time. The azimuth of line VA, say α_A, may be found from the formula

$$\tan \alpha_A = \frac{X_A - X_V}{Y_A - Y_V} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

and α_B and α_C, the azimuths of lines VB and VC, are found by similar formulas. As a check on the computed values of M_A, M_B, and M_C, find the length of VA, by the formula,

$$\overline{VA} = \frac{X_A - XL_L}{\sin \alpha_A} = \frac{Y_A - Y_L}{\cos \alpha_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The lengths VB and VC are determined by similar formulas. Next compute M_A, M_B, and M_C by the formulas:

$$\tan M_A = \frac{\overline{VA}}{Z_L - Z_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26a)$$

$$\tan M_B = \frac{\overline{VB}}{Z_L - Z_B} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26b)$$

and

$$\tan M_C = \frac{\overline{VC}}{Z_L - Z_C} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26c)$$

Elements of Exterior Orientation.—It is now proposed to develop a method of finding the elements of exterior orientation by an application of position circles, thus finding the tilt and the swing of the photograph and the azimuth on the ground-survey system of the principal plane. Thus in Fig. 5, length Lo is the principal distance or (as it is usually called) the focal length f, whereas o is the principal point and v is the vertical point on the photograph. The principal line is ov. In the ground datum plane point V is the nadir point, or vertical point, and O is the intersection of the camera axis with the ground plane. The swing s is the plate azimuth of the principal line ov reckoned clockwise from the positive end of the y-axis on the photograph. Line VO is the intersection of the principal plane with the datum plane or ground plane and its azimuth (α_{VO}) reckoned, say, from the north, is an element of exterior orientation. One of the points of control, A, is photographed at point a and point A′ is the intersection of line LA with the datum plane. Similar points for the control points B and C are shown at b, c, B′, and C′. From calculations already made m_a, m_b, m_c, M_A, M_B, and M_C are known as well as the azimuth angles on the plate α_a, α_b, α_c and these on the ground α_A, α_B, and α_C. The solution of this problem requires the determination of the tilt of the photograph, i, which is equal to the angle oLv or OLV, the swing, s, and the azimuth of the principal plane, α_{VO}.

To facilitate this solution a sphere is taken with its center at the camera station L. Although the radius might be made any arbitrary length, in this case it is taken equal to the focal length. Hence, the plane of the picture will be tangent to the sphere at point o. The various lines and planes through

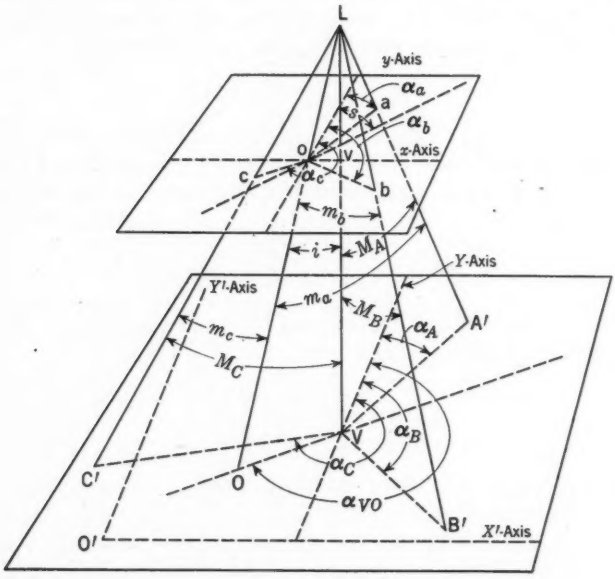

FIG. 5

point L may be considered to intersect the surface of the sphere and thus give points and arcs upon it. As yet the position of v is unknown because it has not been established with respect to point o; nor has the position of point O been established on the ground-control system. It is known, however, that on the surface of the sphere the distances in arc from point o to points a, b, and c as projected onto the sphere by the radial lines are equal, respectively, to m_a, m_b, and m_c and that point V on the sphere must lie at arc distances of M_A, M_B, and M_C from points a, b, and c, respectively. Therefore, if position circles are drawn on the sphere with points a, b, and c, respectively, as centers using in turn the arcs M_A, M_B, and M_C as radii, point V must lie at their intersection. To show these circles requires a projection similar to an azimuthal equidistant map projection with point o as the center, and with points a, b, and c placed in correct azimuthal relationships. The latter are in the plane of the picture and at distances from point o equal to their actual distances in arc from that point on the sphere. Fig. 6 is such a projection to be considered as showing relationships seen from the inside of the sphere. The lines oa, ob, and oc are drawn so as to reproduce their directions on the photograph and their lengths in arc (equal, respectively, to m_a, m_b, and m_c) are laid off to a certain linear scale, as for instance one millimeter equals one degree. Using the same scale, if arcs are struck with a, b, and c as centers and with M_A, M_B, and M_C,

respectively, as radii, these circles will be represented on the projection fairly accurately in the region of particular interest (the region in the vicinity of point o) and will thus fix, by their intersection, approximately the position of the vertical point v (see Fig. 6). However, for small values of the tilt such as are encountered in well-taken vertical photographs, the distance vo is short

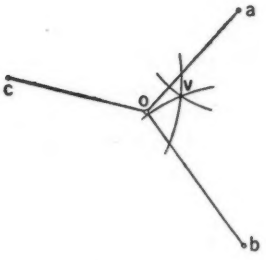

FIG. 6

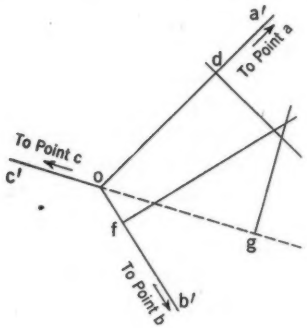

FIG. 7

compared to lengths oa, ob, and oc, and neither its length nor its direction will be well established by this method of construction. For a better determination, the scale of the projection will be much enlarged to show only the area in the vicinity of points o and v and, therefore, with points a, b, and c, no longer available as centers of arcs. Thus in Fig. 7 by use of the coordinates of the points a, b, and c in the photograph, or in some other way, lay off the lines oa′, ob′, and oc′ to represent the lines radiating from point o to points a, b, and c. It is known that the position circle with its center at point a and with its radius of M_A cuts line oa′ at a distance equal to $m_a - M_A$ from point o. This intersection will be on the same side of point o as point a′ if $m_a - M_A$ is plus, but on the opposite side of point o from a′, and, therefore, on a′o extended, if $m_a - M_A$ is minus. Therefore (see Fig. 7) line od is laid off equal to $m_a - M_A$ to a large scale such as 1 mm = 1 min. In this illustrative example, od is laid off on line oa, since it is assumed that M_A is smaller than m_a; and at point d a perpendicular to line oa′ is erected. The perpendicular to od at d is the tangent to the position circle at that point and may be considered to represent approximately the position circle for a short distance on either side of this point. Similarly, line of is laid off along line ob′, and line og along line oc′ extended, equal in length, respectively, to $m_b - M_B$ and $m_c - M_C$. (If the difference such as $m_a - M_A$ is plus, it is laid off from point o in the direction of the point; whereas if it is minus, as in the case of line og, it is laid off in the opposite direction or on the line extended.) Perpendiculars to lines ob′ and oc′ are drawn at points f and g to represent the tangents to the position circles through those points. While the position circles, if they could be accurately drawn, would meet in a point, the perpendiculars will meet in pairs to give three points of intersection and what may be considered a triangle of error. By considering that the arcs would lie on the same side of the tangents that the control points

do, one may spot the approximate location of point v. Then, by scaling off the length of line ov and its direction with respect to the y-axis, values of i and s may be obtained.

By a small amount of computation and further construction work, an accurate location of point v may be obtained. Thus it is known that for some little distance from the point of tangency of a circle the offset from the tangent to the circle varies closely as the square of the distance along the tangent; and is equal to $h^2/(2\,r)$, in which h is the distance from the point of tangency and r the radius of the circle. In this case the distance from the approximate location of point v to the tangent point may be scaled and the radius is M_A to the scale of the drawing, assumed herein to be 1 mm = 1 min. After the offsets have been computed, they may be laid off from their respective tangents in the vicinity of point v, care being taken to lay them off on the proper side of the tangent, and parallels to the tangents drawn through the extremities of the offsets. These should meet in a point or, if not, leave a very small triangle of error to fix point v in either case very closely. By scaling, the tilt should then be found to within a small fraction of a minute and the swing s to within a small fraction of a degree.

By considering the construction to be in the ground plane with the sphere tangent at the vertical point, a similar construction centering on point V gives a check solution for i and makes it possible to scale off the value of α_{VO}, which should be as accurate as the value previously found for s.

Mathematical Solution.—Of course graphical methods may be avoided in finding i, s, α_{ov}, and α_{VO} by using a mathematical method based on the application of spherical trigonometry. Consider the lines Lo, Lv, La, Lb, and Lc to be prolonged from the center of the sphere at point L, Fig. 5, until they intersect the surface of the sphere; and suppose these points to be connected with arcs of great circles to form the construction sketched in Fig. 8. The arcs ab,

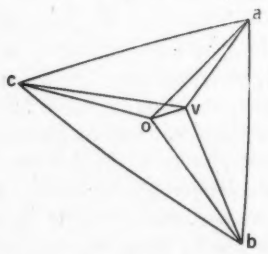

Fig. 8

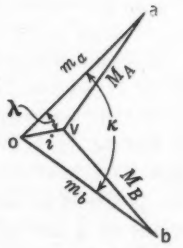

Fig. 9

bc, and ac are equal, respectively, to β_{ab}, β_{bc}, and β_{ca}, the angles at the common vertex of the plate and the ground pyramid. Solving this triangle, formed by points a, b, and c, one can find its interior angles. Also arcs oa, ob, and oc are equal, respectively, to the known angles m_a, m_b, and m_c. Angles aob, boc, and aoc can be found from the plate azimuths. By the sine law one may then solve by spherical trigonometry to find the angles at points a, b, and c in the triangles aob, boc, and aoc. Similarly, knowing that arcs av, bv, and cv are equal,

respectively, to M_A, M_B, and M_C, and finding the angles avb, bvc, and avc from the azimuths on the ground system of lines VA, VB, and VC, one may solve for the angles at points a, b, and c, in the triangles avb, bvc, and avc. Then in the triangle avo, for instance, the sides av and ao are known; the angle vao is the difference between angles bav and bao. A solution may then be made for the length vo or i and for the angles at points v and o. From these angles the azimuth of line ov, or s, can be found by adding the angle aov to the plate azimuth of line oa; and the azimuth of line vo on the ground-survey system may be obtained by subtracting the angle avo from the azimuth of line VA on the ground system. The solution may be checked by similar use of the triangles bvo and ovc, one or both. These calculations are rather long and tedious and are not likely to check very well unless extended to a number of decimal places either by natural functions or by logarithms, since the angles at points a, b, and c in the triangles oav, obv, and ovc are small.

The following method may be adopted to check the values of i, s, and α_{VO} and to obtain them to a higher degree of precision. Consider two of the three triangles involving the line vo (say, aov and bov with the side vo or i, Fig. 9, in common). The sides ao, bo, av, and bv are known accurately to be equal to m_a, m_b, M_A, and M_B, respectively. The value of $\overline{vo}$ is known approximately from the graphical solution. For simplicity let the angle boa, which is known, be designated by κ and the angle aov, which is known only approximately, be designated by λ; then:

$$\cos M_A = \cos m_a \cos i + \sin m_a \sin i \cos \lambda \ldots\ldots\ldots\ldots (27a)$$

and

$$\cos M_B = \cos m_b \cos i + \sin m_b \sin i \cos (\kappa - \lambda) \ldots\ldots\ldots (27b)$$

By solving each equation for $\sin i$ and equating values,

$$\sin i = \frac{\cos M_A - \cos m_a \cos i}{\sin m_a \cos \lambda} = \frac{\cos M_B - \cos m_b \cos i}{\sin m_b \cos (\kappa - \lambda)} \ldots\ldots (28a)$$

which may be written

$$\frac{\cos (\lambda - \kappa)}{\cos \lambda} = \frac{\sin m_a (\cos M_B - \cos m_b \cos i)}{\sin m_b (\cos M_A - \cos m_a \cos i)} \ldots\ldots\ldots\ldots (28b)$$

Since $\cos (\lambda - \kappa) = \cos \lambda \cos \kappa + \sin \lambda \sin \kappa$:

$$\cos \kappa + \tan \lambda \sin \kappa = \frac{\sin m_a (\cos M_B - \cos m_b \cos i)}{\sin m_b (\cos M_A - \cos m_a \cos i)} \ldots\ldots\ldots (28c)$$

Solving for $\tan \lambda$:

$$\tan \lambda = \left[\frac{\sin m_a (\cos M_B - \cos m_b \cos i)}{\sin m_b (\cos M_A - \cos m_a \cos i)} - \cos \kappa \right] \frac{1}{\sin \kappa} \ldots\ldots (29)$$

From the graphical solution, i is known approximately. Therefore, since i is small and the cosine changes but slowly for changes in the angle, $\cos i$ may be considered to be known closely. All other quantities involved in the right-hand

member of Eq. 29 are known; and by substitution in this equation an approximate value of λ can be determined, a value accurate enough for all practical purposes. Eqs. 28 can then be solved for i from its sine, giving a close approximation to the true value of i. The swing s may be obtained by adding λ to the direction of line ao. By the sine law the angle avo may be found, and this value, subtracted from the azimuth of line VA, will give α_{VO}. Thus, i, s, and α_{VO} may be determined to a high degree of precision by computations supplementing the graphical solution.

Azimuths from Measurements of Photographs.—Another application of "position circles" is in finding azimuths from the vertical point on the ground as functions of directions from the principal point to points on the photograph. If this is done for a point showing in the overlapping part of two photographs whose elements of orientation have been found, the data may be used to compute the coordinates and elevation of the point. Thus, let position q be any point on a vertical photograph. Suppose that the coordinates of this point on the photograph are x_q and y_q. The direction of line q measured on the photograph may then be scaled, or computed from the relation that

$$\tan \alpha_q = \frac{x_q}{y_q} \qquad (30)$$

Solve also for the angle m_q at the vertex of the pyramid.

On the diagram from which line ov, or i, was originally determined, or on a new diagram such as Fig. 10, lay off the line oq′ in the direction of point q,

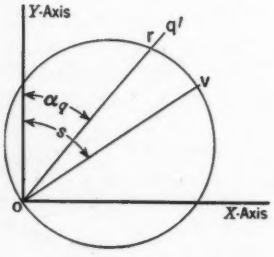

FIG. 10

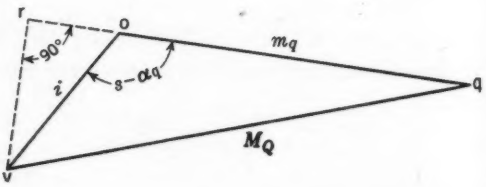

FIG. 11

making the angle between line oq′ and the y-axis equal to α_q. Also draw the line ov (if it has not already been drawn), laying off its length to the same scale that was formerly used in determining line ov or using a similar large scale such as 1 mm = 1 min or arc. If many points are to be dealt with, a circle should now be constructed on line ov as a diameter. The chord formed by the intersection of the line (prolonged if necessary) and the circle (line or in Fig. 10) is scaled, as is also the chord rv—say, in minutes of arc to the same scale as line ov. The distance ro gives the approximate difference between M_Q and m_q. A more accurate value of this difference may be found by correcting by an amount equal to $\frac{\overline{vr}^2}{2\,M_Q}$ in which $\overline{vr}$ and M_Q are both expressed in minutes of arc. The correction in this case should be added to $\overline{ro}$ to derive a more accurate value

of the difference $M_Q - m_q$. By adding this difference to m_* the distance M_Q may be found. In the triangle voq, Fig. 11,

$$\frac{\sin v}{\sin (s - \alpha_q)} = \frac{\sin m_q}{\sin M_Q} \quad \ldots\ldots (31a)$$

and

$$\frac{\sin q}{\sin (s - \alpha_q)} = \frac{\sin i}{\sin M_Q} \quad \ldots\ldots (31b)$$

The angles v and q can be determined by Eqs. 31.

An alternative solution, which may be made with the slide rule or computing machine, is to consider the right triangle rvq, Fig. 11, and to use the formula—

$$\sin q = \frac{\sin \overline{rv}}{\sin M_Q} \quad \ldots\ldots (32)$$

—to find the value of q. The spherical excess, in minutes, (e) of triangle voq may be found approximately by using the formula,

$$e = \frac{\pi}{21{,}600} \times \overline{rv} \times \overline{mq} \quad \ldots\ldots (33)$$

in which $\overline{rv}$ and m_q are to be expressed in minutes of arc. Then $<$ ovq is the difference between $(180 + e)$ and $(s - \alpha_q + <$ ovq$)$. The azimuth of the arc Vq is obtained by adding the angle ovq to the azimuth α_{vo} of arc VO. If a map is under construction, after the vertical points have been plotted, radial lines to points such as q can be laid off with the protractor, or the coordinates of points suitably placed for intersections can be computed. By scaling or computing, the horizontal distance from the located points to the camera stations can be found and, by using angles such as M_Q, the elevation may be determined either graphically or by calculation. For the point q the difference in elevation between it and the camera station is equal to the horizontal distance times the cotangent of M_Q. This completes the determination of the coordinates of point q.

Numerical Example

To demonstrate the application of theory by a numerical example, a 7-in. by 9-in. photograph is selected for the purpose of determining the elements of orientation. This print designated ART-3–14 was one of a series taken for the Agricultural Adjustment Administration, U. S. Department of Agriculture, which covers an area well controlled by ground surveys. On this photograph three control points A, B, and C, well located for the purpose, were chosen; their coordinates, referred to axes through the principal point (center) of the photograph, were measured by a method that was fairly precise. These coordinates, x and y, were determined to hundredths of a millimeter. The ground coordinates came from triangulation and traverse surveys for the X and Y, and from spirit leveling for the Z. They are expressed in feet, with the Z-coordinates giving the elevations above sea level, as shown in Table 1. The

lengths of AB, BC, and CA, and the horizontal and slope distances in Table 1, were computed by aid of a computing machine from the survey coordinates by use of the formulas in the first part of this paper (see Eqs. 7 and 8).

TABLE 1.—COORDINATES OF CONTROL POINTS; HORIZONTAL AND SLOPE DISTANCES

Description	Point A	Point B	Point C
Ground Coordinates, in Feet:			
X	35,900.2	27,420.5	36,541.0
Y	5,890.9	15,924.9	16,481.3
Z (elevations above sea level)	1,238.4	1,384.6	1,348.9
Map Coordinates, in Millimeters:			
x	−26.88	+102.69	−33.11
y	+74.57	− 70.70	−83.16
Horizontal distances, in feet	13,137.23	9,137.46	10,609.77
Slope distances, in feet	13,138.03	9,137.53	10,610.34

Calculations are now made from the coordinates of the points on the photograph and the focal length f, which is known to be 209.55 mm, to determine the azimuths (α in line 2, Table 2) from the principal point to the control points.

TABLE 2.—AZIMUTHS FROM PRINCIPAL POINT TO CONTROL POINTS; ANGLES AT VERTEX

Line	Quantity	Point a	Point b	Point c
1	Tan $\alpha = x/y$	0.36046668	1.45247525	0.39814815
2	Azimuth, α	340° 10′ 39″	124° 32′ 48″	201° 42′ 36″
3	$x^2 + y^2$	6,283.2193	15,543.7261	8,011.8577
4	Distance, $\sqrt{x^2 + y^2}$	79.267	124.675	89.509
5	$x_2 - x_1$	+129.57	−135.80	+6.23
6	$y_2 - y_1$	−145.27	− 12.46	+157.73
7	$l^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$	37,891.7578	18,596.8916	24,917.5658
8	Length, l	194.655	136.370	157.853
9	$x^2 + y^2 + f^2$	50,194.42	59,454.93	51,923.06
10	Slope distance ($\sqrt{}$ line 9)	224.041	243.834	227.866
11	Sin m	0.353806	0.511311	0.392814
12	Tan m	0.378272	0.59465	0.427149
13	Angle m	20° 43′ 13″	30° 45′ 04″	23° 07′ 47″

The distances ab, bc, and ca on the picture, and the slope distances La, Lb, and Lc, are given in lines 7 and 10, Table 2. The angles between Lo and each of the lines La, Lb, and Lc, in turn, are given in line 13, Table 2. As a check, these angles are found from their tangents and checked by their sines.

The vertex angles aLb, bLc, and cLa, Fig. 2, are now computed by applying Eqs. 5, in turn, to each of the three triangles aLb, bLc, and cLa, the sides of which are known. The foregoing calculations do not differ greatly from those used by Professor Church[4]. Thus, by Eq. 5*a*:

$$\cos \beta_{ab} = \frac{-37{,}891.7578 + 50{,}194.4218 + 59{,}454.9286}{2 \times 224.041 \times 243.834}$$

$$= \frac{71{,}757.5926}{109{,}257.6264} = 0.65677422;$$

$\beta_{ab} = 48° 56' 44''.46$; and sin $\beta_{ab} = 0.75408730$. Similarly, by Eq. 5*b*, cos $\beta_{bc} = 0.83494086$; $\beta_{bc} = 33° 23' 25''.15$; and sin $\beta_{bc} = 0.55033968$. Finally, by Eq. 5*c*, cos $\beta_{ca} = 0.75610098$; $\beta_{ca} = 40° 52' 42''.00$; and sin $\beta_{ca} = 0.65445494$.

The next step in the solution is to find values of LA, LB, and LC, and for this purpose it is necessary to have an approximate value of one of these, say, of LA = ρ_A, to use in the solution. This approximate value may be found in any one of several ways. A simple one is to consider the lines ab, bc, and ca to be parallel, respectively, to AB, BC, and CA, and to solve by proportion considering the face triangles to be similar. In this example a somewhat more precise method, partly graphical and partly by calculation, which will not be described, was used. The approximate value of LA found by this method was 15,234 ft. Using this value the calculations shown below were made. Referring to Fig. 3: AB = L_{AB} = 13,138.03; BC = L_{BC} = 9,137.53; CA = $L_{CA'}$ = 10,610.34. Substituting these values of L and β in Eqs. 9: Sin $\angle$ (2) = 0.87439016 and $\angle$ (2) = 60° 58′ 22″.38; $\angle$ (1) = 180° − β_{ab} − $\angle$ (2) = 70° 04′ 53″.16; and sin $\angle$ (1) = 0.94017777. By Eq. 9*c*, ρ_B = 16,380.18. By similar adaptations of Eqs. 9 values of ρ_C and $\rho_{A'}$ are found to be 15,169.91 and 15,213.49, respectively. As this is done, angles 3, 4, 5, and 6 are found to be, respectively, 66° 00′ 58″.40, 80° 35′ 36″.45, 69° 46′ 52″.82, and 69° 20′ 25″.18. By substitution of values of the cosines of angles 1 to 6 in Eq. 13 it is possible to compute u as $-0.7021 \times 2.4873 \times 0.9795 = -1.7105$, which, by Eq. 15*a*, is equal to $\frac{\Delta\rho_{A'}}{\Delta\rho_A}$.

As stated in connection with Eq. 15*b*, $\Delta\rho_{A'}$ and $\Delta\rho_A$ are the corrections to be applied to $\rho_{A'}$ and ρ_A, respectively, to make them equal; thus, 15,234 + $\Delta\rho_A$ = 15,213.49 + $\Delta\rho_{A'}$ and $\Delta\rho_A = \Delta\rho_{A'} - 21.51$. As computed by Eq. 15*a*, $\Delta\rho_{A'} = -1.7105\,\Delta\rho_A$; and, thus, $2.7105\,\Delta\rho_A = -21.51$ and $\Delta\rho_A = -7.567$. Also $\Delta\rho_B = -0.7021\,\Delta\rho_A = +5.313$, $\Delta\rho_C = +1.7463\,\Delta\rho_A = -13.214$, and $\Delta\rho_{A'} = -1.7105\,\Delta\rho_A = +12.943$. Applying these corrections to the previously computed values of ρ, the following corrected values of these quantities are: ρ_A = 15,226.43, ρ_B = 16,385.49, ρ_C = 15,156.70, and $\rho_{A'}$ = 15,226.43.

A recalculation of these quantities (which is not shown herein for lack of space) showed that these results were practically correct, the value of ρ_B checking exactly, that of ρ_C within a tenth, and that of $\rho_{A'}$ within two one hundredths. These values have been computed to more significant figures than would be warranted in practice in order to show that the solution may be extended to a high degree of precision by the method outlined.

To set up the equations for solving for the coordinates in space, X_L, Y_L, and Z_L of the camera station, it is necessary to have approximate values of these coordinates. A graphical solution of the three-point problem, under the assumption that the photograph is vertical, gives approximate values of X_L and Y_L. The third coordinate, Z_L, may be found by use of the m-angles, and values of ρ to obtain three values of Z_L, which are then averaged to derive the value to be used. In this case a somewhat more accurate method was used but will not be explained.

The approximate values used and the resulting equations and solution for finding the corrections to these are presented in Table 3(*a*).

TABLE 3.—Illustrative Example; Tabulation of Calculations

Line	Quantity	Point A	Point B	Point C
	(*a*) Solution of Eqs. 18 (X_L = 34,456.0, Y_L = 11,249.5, and Z_L = 15,419.8)			
1	$X_L - X$	− 1,444.2	+ 7,035.5	− 2,085.0
2	$Y_L - Y$	+ 5,358.6	− 4,675.4	− 5,231.8
3	$Z_L - Z$	+14,181.4	+14,035.2	+14,070.9
4	ρ	15,226.43	16,385.49	15,156.59
5	$(X_L - X)^2$	2,085,713	49,498,260	4,347,225
6	$(Y_L - Y)^2$	28,714,594	21,859,365	27,371,731
7	$(Z_L - Z)^2$	201,112,106	196,986,839	197,990,227
8	Σ	231,912,413	268,344,464	229,709,183
9	$\Sigma\rho^2$	231,844,170	268,484,282	229,722,220
10	Difference	+68,243	−139,818	−13,037
	(*b*) Solution of Eqs. 23, Using Corrected Coordinates, X_L = 34,463.2, Y_L = 11,245.2, Z_L = 15,419.7			
1	$X_L - X$	− 1,437.0	+ 7,042.7	− 2,077.8
2	$Y_L - Y$	+ 5,354.3	− 4,679.7	− 5,236.1
3	$Z_L - Z$	+14,181.3	+14,035.1	+14,070.8
4	ρ	15,226.43	16,385.49	15,156.59
11	Cos M	0.931361	0.856561	0.928362
12	Angles M	21° 21′ 07″	31° 04′ 03″	21° 49′ 09″
	(*c*) Position of the Vertical Point v on the Photograph and of Point v′ on the Map			
	Angle m:			
13	Degrees	20° 43′ 12″	30° 45′ 04″	23° 07′ 47″
14	Minutes	1,243.2	1,845.1	1,387.8
	Angle M:			
15	Degrees	21° 21′ 07″	31° 04′ 03″	21° 49′ 09″
16	Minutes	1,281.1	1,864.0	1,309.1
	$m - M$:			
17	Degrees	−0° 37′ 55″	−0° 18′ 59″	+1° 18′ 38″
18	Minutes	−37.9	−19.0	+78.6
	Ground Coordinates:			
19	h (min)	79	85	34
20	Offset, $h^2/(2\,M)$	2.4	2.0	0.4
	Map Coordinates:			
21	h (min)	77	84	38
22	Offset, $h^2/(2\,m)$	2.4	1.9	0.5

TABLE 4.—Computation of Corrections by Eqs. 22

Eq.	ΔX	ΔY	ΔZ	k	Constant
A	−1,444.2	+5,358.6	+14,181.4	+1	+34,122
B	+7,035.5	−4,675.4	+14,035.2	+1	−69,909
C	−2,085.0	−5,231.8	+14,070.9	+1	− 6,519
1	−0.10184	+0.37786	+1	+0.000071	+ 2.4061
2	+0.50128	−0.33312	+1	+0.000071	− 4.9810
3	−0.14818	−0.37182	+1	+0.000071	− 0.4633
4	+0.60312	−0.71098	0	0	− 7.3871
5	+0.64946	+0.03870	0	0	− 4.5177
6	+1	−1.17885	0	0	−12.2482
7	+1	+0.05958	0	0	− 6.9562
8	0	+1.23843	0	0	+ 5.2920

The equations A, B, and C, Table 4, are the original Eqs. 22 from which Eqs. 1, 2, and 3 are derived by dividing each in turn by its ΔZ-coefficient. Eq. 4, Table 4, is obtained by subtracting Eq. 1 from Eq. 2; and Eq. 5 is similarly obtained by subtracting 3 from 2. This eliminates ΔZ. A similar elimination of ΔX gives Eq. 8, which may be solved for ΔY. Substitutions in Eqs. 6 and 1 of Table 4 give ΔX and ΔZ; thus: $\Delta Y = \frac{-5.2920}{1.23843} = -4.273$; $\Delta X = +12.248 - 5.037 = +7.211$; and $\Delta Z = -2.406 + 1.615 + 0.734 = -0.057$.

Since k is equal to $\frac{1}{2}(\Delta X^2 + \Delta Y^2 + \Delta Z^2)$ these terms are negligible in this case, and have been neglected. By making a solution without them and then computing a value of k to be substituted, a fairly exact solution is possible even when ΔX, ΔY, and ΔZ are large.

By applying the corrections just found to the approximate values of the coordinates, the following final values are found for the camera station L: $X_L = 34{,}463.2$; $Y_L = 11{,}245.2$; and $Z_L = 15{,}419.7$.

The angles, M_A, M_B, and M_C, which the lines LA, LB, and LC make with the vertical, are computed by Eq. 23, as shown in Table 3(*b*). The values of X_L, Y_L, and Z_L may be checked as was done in this case by applying the formulas for computing distances from coordinate difference (Eqs. 7 and 18) to obtain values of LA, LB, and LC from the quantities in Table 3(*b*). These calculations are not shown.

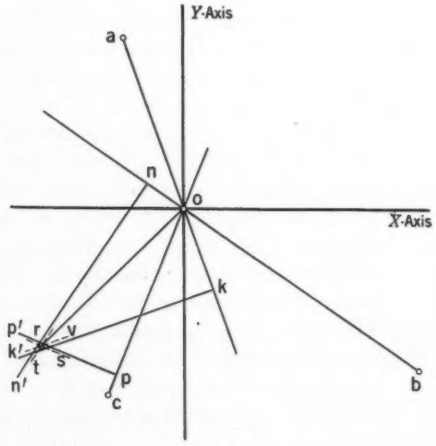

FIG. 12

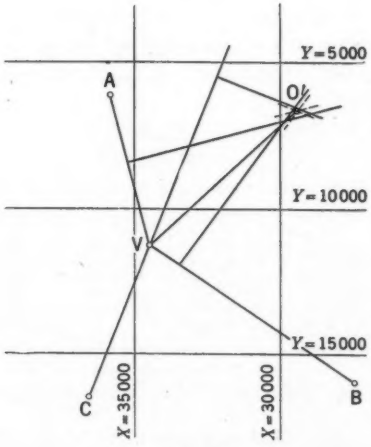

FIG. 13

The vertical point of the photograph is now found in Fig. 12, which is drawn similarly to Fig. 7. The distances ok, on, and op are laid off equal to $m_a - M_A$, $m_b - M_B$, and $m_c - M_C$, respectively, using a scale of 1 mm = 1 min of arc. In Fig. 12, the three perpendiculars kk′, nn′, and pp′ instead of meeting in a point form, by their intersections, a triangle rst that may be called a triangle of error or, more accurately, the "triangle of the tangents." It is known that v, the point sought, lies in or near this triangle in such a position that its distances

from the perpendiculars kk′, nn′, and pp′ are proportional to the squares of the distances from the lines oa, ob, and oc, respectively. With this in mind a point was spotted as a trial position of v. (This is not shown in Fig. 12 but was near the position shown for v.) By the method previously explained these offsets are calculated and laid off from their respective lines to locate some parallel lines, shown dotted, which should meet at the required point, v. In this case a small triangle of error was found, within which the point v was located by estimation. The distance ov was scaled and found to be 86.8 min or 1° 26′.8, which is the tilt i. The direction of ov, clockwise from the y-axis, was found by scaling to be 225° 55′. This is the swing.

The azimuth of the principal plane is found by aid of Fig. 13, which represents a map upon which the positions of A, B, C, and V are plotted by their coordinates to a scale of 1 in. = 1,000 ft. By a construction similar to that in Fig. 12, the point O′ where the camera axis prolonged would meet the map plane is found. The scaled length O′V gives the tilt, and the azimuth of VO′ (from the south in this case) gives the azimuth of the principal plane. The value of the tilt was found to be 1° 26′.8 (checking that previously determined) and the azimuth of the principal plane was found to be 227° 40′.

As an indication of the precision of the method of Table 3(*c*) it may be stated that an analytical method gave the following values: $i = 1° 26′ 47″$; swing, $s = 225° 59′ 35″$; and the azimuth of the principal plane, $\alpha_{VO} = 227° 24′ 23″$. Thus the tilt checked exactly, the swing within 5 min, and the azimuth of the principal plane within 15 min.

Eq. 29 will now be applied to find more exact values of the swing and of the azimuth of the principal plane, and Eq. 28*a* to obtain a more exact value of the tilt. The use of both of these equations affords a partial check on the numerical work.

TABLE 5.—DATA FOR THE SOLUTION OF EQ. 29

Symbol	Angle	Sine	Cosine
m_a.....	23° 07′ 47″	0.392814	0.919618
M_A.....	21° 49′ 09″	0.371678	0.928362
m_b.....	20° 43′ 13″	0.353806	0.935319
M_B.....	21° 21′ 07″	0.364096	0.931361
κ.....	138° 28′ 03″	0.663045	−0.748580
i.....	1° 26′ 48″[a]	....	0.999681

[a] Approximately

It is known that line ov lies between lines oc and oa in this case; and, therefore, κ is the angle between the latter two lines, or is equal to 138° 28′ 03″, the difference between the plate azimuths of the lines oc and oa, previously calculated. The angle λ is measured clockwise from the line oc. In applying Eqs. 28*a* and 29 the subscripts *a*, *b*, *A*, *B* are to be construed as applying to the points that hitherto have been called, respectively, c, a, C, and A. Substituting, in Eq. 29, appropriate values from Table 5:

$$\tan \lambda = \left[\frac{0.392814\,(0.931362 - 0.935021)}{0.353806\,(0.928362 - 0.919325)} + 0.748580 \right] \frac{1}{0.663045} = 0.451161.$$

Therefore: $\lambda = 24° 16′ 59″$; $\sin \lambda = 0.411245$; $\cos \lambda = 0.911525$; $\kappa - \lambda = 114° 11′ 04″$; and $\cos (\kappa - \lambda) = -0.409675$. Similarly, for the solution of Eq. 28*a*: By the first equality—

$$\sin i = \frac{0.009037}{0.358060} = 0.025239 \text{ and } i = 1° 26′ 47″$$

and, by the second equality—

$$\sin i = \frac{-0.003659}{-0.144945} = 0.025244$$

The latter value checks the former value, although the first of the two expressions in Eq. 28*a* is probably the more exact since it is derived from larger quantities. The swing is found by adding the angle λ to the plate azimuth of line oc and is thus found to be 225° 59′ 35″.

It remains to find the azimuth of the principal plane. This may be done by solving for the angle at point v in the triangle ocv by using the sine law and then subtracting this angle from the azimuth of line VC. The latter azimuth is found by the coordinate differences between the *X*-ordinate and *Y*-ordinate of the vertical point V and those of point C. These coordinates of point V are the same as those of the camera station L.

Thus, $\sin \overline{ovc} = \frac{\sin \lambda \sin m_a}{\sin M_A} = 0.434631$ and $\angle \overline{ovc} = 154° 14' 17''$. Tan $\alpha_{VC} = \frac{X_C - X_V}{Y_C - Y_V} = \frac{+2{,}077.8}{+5{,}236.1} = 0.396822$ and α_{VC}, the azimuth of line VC, = 21° 38′ 40″. By subtracting the angle ovc from α_{VC} the azimuth of the principal plane is found to be 227° 24′ 23″. Thus, as previously stated, the swing found by this more exact method checks the value found by the graphical method to 0.1 min, the swing checks within 5 min, and the azimuth of the principal plane within 15 min.

Summary

It is believed that the main contribution of this paper lies in the use of the cosines of the base angles of the ground pyramid to obtain a rapid solution in finding the lengths of the edges of this pyramid, and in the application of "position circles" in fixing the position of the vertical point, first on the photograph and then on the map. Although the method of solving equations of the second degree such as those arising in finding the coordinates of the camera station by approximations, using Taylor's theorem as a basis, is well known, it is thought that the use of a constant *k* to represent the sum of the squares of the corrections is an innovation. The special application of the methods of spherical trigonometry to find the tilt, the swing, and the azimuth of the principal plane, it is believed, also has some merit.

APPENDIX

Notation

The following letter symbols are introduced in this paper. Discussers are requested to adapt their comments to these letters to avoid confusion of concept. In general, ground-point locations and line designations are not italicized, single letters in italics being used to designate quantities that can take a numerical equivalent. In designating points or lines small letters refer to

locations on the map or photograph and capital letters are the corresponding points or lines on the ground

a = a mathematical coefficient (with b and c in Eqs. 20, 21, and 22);
b = a coefficient (see a);
c = a coefficient (see a);
e = spherical excess;
f = focal length of a camera;
h = a distance along a tangent to a circle, measured from the point of tangency;
i = tilt angle of a photograph;
k = a substitution constant (see Eq. 22);
L = length between traverse points measured on the ground;
l = length between traverse points measured on the map or photograph, corresponding to L;
M = angle to any ground object from a vertical through the lens center (see Fig. 5);
m = angle to any map or photograph object, measured from the plate perpendicular, corresponding to M (see Fig. 1);
q = a substitution constant (see Eq. 22);
r = radius of a circle;
s = swing, or angle made by the principal line, with the Y-axis;
u = a constant (see Eq. 12);
X = variable distances parallel to the X-axis, as observed on the ground; ΔX = correction to be applied to an X-value;
x = variable distances on a map or photograph corresponding to X;
Y = variable distances parallel to the Y-axis, as observed on the ground; ΔY = correction to be applied to a Y-value;
y = variable distances on a map or photograph corresponding to Y;
Z = variable value of height or ground elevation; ΔZ = correction to be applied to an elevation;
α = azimuth from the y-axis in the xy-plane, or from the Y-axis in the XY-plane, the direction line being denoted by appropriate subscript;
β = face angles of a triangular pyramid, the particular face being shown by appropriate subscript;
κ = an angle boa (Fig. 8) in a plane that is tangent to a sphere at point o;
λ = an angle aov (Fig. 8) in a plane that is tangent to a sphere at point o; and
ρ = radial distances from the lens center to ground points designated by appropriate subscripts.

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PAPERS

RECHARGE AND DEPLETION OF GROUND-WATER SUPPLIES

By Charles L. McGuinness,[1] Esq.

Synopsis

It is widely believed that the water table is declining and that ground-water supplies are being depleted because of deforestation, drainage, improper methods of land cultivation, and pumping from wells. In addition, many fantastic explanations are offered to account for depletion of ground-water supplies. The paper describes the nature of ground-water reservoirs or aquifers, their recharge from precipitation and stream flow, and their discharge through seeps and springs and by evaporation and transpiration. It shows that changes in ground-water levels are caused by climatic changes or by human activities that affect the rates at which the ground-water reservoirs are recharged and discharged. Although additional investigation is needed, there is no reason to believe that changes in ground-water levels due to deforestation, drainage, and land cultivation have been progressive over long periods in most parts of the United States. The most pressing problems are those resulting from the withdrawal of water from wells. The paper divides the productive aquifers into two broad classes—those with high rates of recharge and large perennial supplies and those with low rates of recharge and small perennial supplies—and describes numerous examples of aquifers of both types. Artificial recharge as a method of conserving ground water is described briefly and is shown to be both practicable and necessary at certain places. The paper concludes that at some places the present rates of withdrawal from wells are excessive and will have to be curtailed sooner or later, but at many other places the rate of recharge is high and large supplies will be perennially available to wells.

Introduction

A persistent belief has arisen that the water table is declining progressively because of deforestation, drainage, improper methods of land cultivation, and

Note.—Written comments are invited for immediate publication; to insure publication the last discussion whould be submitted by February 1, 1947.

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pumping from wells. The growth of such a belief has been especially rapid since 1930, and is particularly strong in the Middle West. Ground-water levels may decline as the result of deficient or poorly distributed precipitation, pumping from wells, installation of drainage ditches, or other factors. The causes of a decline in ground-water levels at any particular place are generally so complex that attempts to define them without careful investigation may lead to confusion. Thus, it is to be expected that a large number of people might believe that ground-water supplies are diminishing to such an extent that sooner or later they will be largely depleted. It is obviously impossible to present this large and complicated subject in a single paper, but an attempt will be made to give some information on the sources of ground water, the extent of depletion, and the prospects for continuing supplies. General discussions of these phases of ground-water hydrology are found in several papers by O. E. Meinzer (1*a*)(2)(3)(4)(5)(6)(7).[2]

Recharge and Discharge of Ground-Water Reservoirs

A discussion of recharge and depletion of ground-water supplies involves the concept of ground-water reservoirs. The rock formations of the earth—which include both consolidated rocks such as sandstone and limestone and unconsolidated sand and gravel—constitute great natural ground-water reservoirs. These reservoirs, or aquifers, are replenished by water derived directly from rain and snow or from streams, and they are discharged under natural conditions through seeps and springs and by evaporation and transpiration of plants. The water levels in the subterranean reservoirs fluctuate seasonally and with cycles of drought and excessive precipitation. Both recharge and discharge are high in wet years, and the water table is high. In dry years recharge is diminished, water is withdrawn from storage, the water table declines, and the discharge diminishes. In a sense, the water then remaining in the reservoir is in dead storage, but some of it can always be made available by wells.

The storage capacity of the major subterranean reservoirs is very great, and therefore in times of need, such as periods of drought or other emergencies, quantities of water far in excess of the current recharge, or of the perennial safe yield, can be withdrawn from storage for a few years. However, all heavily pumped aquifers should be thoroughly investigated to determine the rates at which given installations of wells can be pumped indefinitely, so that estimates can be made of the extent to which emergency rates of pumping may have to be reduced.

Under natural conditions, the aquifers are in an approximate state of dynamic equilibrium, although the water table fluctuates seasonally and from year to year. The utilization of ground water by means of wells is an additional discharge imposed on the natural system of recharge and discharge, and upsets the equilibrium (8). Before equilibrium can again be reached, the new discharge must be balanced by an increase in the rate of recharge or by a decrease in the rate of natural discharge, or by a combination of the two. Until a new equilibrium is reached, water must be pumped from storage with a consequent

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

decline in water levels. Ground-water reservoirs, or aquifers, differ widely in areal extent, opportunity for recharge, ability to transmit and store water, and conditions of natural discharge. The properties of the individual aquifers determine how much water can be withdrawn from them by wells and the relative extent to which withdrawal from storage, increase in recharge, and decrease in natural discharge are involved.

A decline in water levels caused by withdrawal of water from wells increases the hydraulic gradient from the areas of recharge and reduces the gradient toward the areas of natural discharge. A decline of the water table in the areas of recharge may result in an increase in the rate of recharge, but only if there is "rejected recharge" (8). Recharge is rejected if the potential rate of recharge exceeds the rate at which water can flow laterally through the aquifer, so that the water table stands at or near the surface and water is disposed of by evaporation and transpiration and by flow into streams. Lowering the water table in such an area reduces the rate at which water is lost, and thus increases the rate of recharge. If the rate of recharge is normally less than the rate at which the aquifer can carry the water away laterally, recharge is not rejected and the rate of recharge is governed (*a*) by the rate at which water is made available by precipitation or by the flow of streams, or (*b*) by the rate at which water can move vertically downward through the soil to the water table and thus escape evaporation. Lowering the water table by withdrawing water from wells does not cause an increase in the rate of recharge in such an area.

A decline of the water table in the areas of natural discharge caused by withdrawal of water from wells reduces the discharge by evaporation and transpiration and also by seepage into streams. Water that is derived not from a loss in storage but from increased recharge or decreased discharge has been termed "salvage" (9).

Over a long period under natural conditions, depletion of the water supply of an aquifer can occur only as the result of a climatic change or a geologic process that causes a reduction in recharge or an increase in discharge. For example, a reduction in recharge might be caused by a blanket of loess deposited by wind over the recharge area of a permeable formation, or by deposition of silt in the bottom of a stream that contributes water to an aquifer. An example of a process that might cause an increase in discharge would be the lowering, by downcutting, of the level of a stream into which ground water is discharging. Depletion of the supply can occur as the result of the activities of man, who not only withdraws water from wells but also performs other actions that may affect the rates of recharge and discharge.

A "Symposium on Fluctuations of Ground-Water Levels" that appeared in 1936 (1) contains papers giving long-time records of ground-water levels at several places in the United States and one place in England. Some of these records illustrate the principle that, under natural conditions, ground-water levels fluctuate with the amount and distribution of precipitation. The well whose hydrograph is shown in Fig. 1 is in the northeastern tip of Oneida County, New York (1*b*), and penetrates glacial sand and gravel. The observations were made by the Black River Regulating District. The graph shows no progressive decline in water level during the period of record. The graph of the water level

in this well (Fig. 1(*a*)) in the latter part of 1944 was about the same as in the latter part of 1935, although before the end of 1944, because of drought, the water level declined a little lower than in 1935.

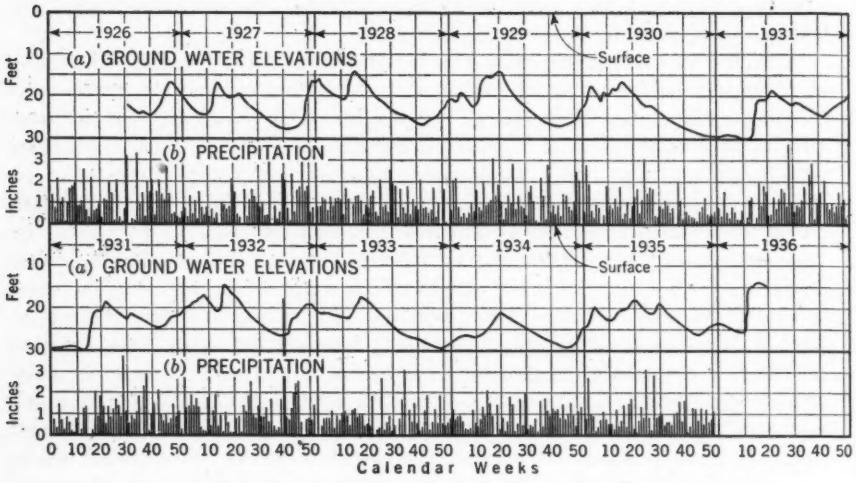

FIG. 1.—HYDROGRAPH OF WELL 6 AT WOODGATE, N. Y., 1926–1936

Fig. 2 illustrates a feature of ground-water recharge that is typical of some areas, particularly in the West, in which the pumpage from irrigation wells is in excess of the rate of recharge in years of normal or low precipitation, resulting in withdrawal of water from storage. However, the water is replenished during

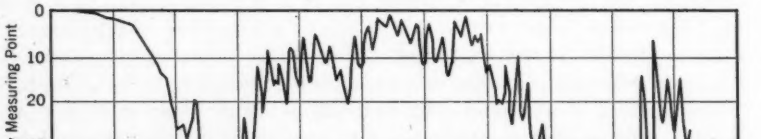

FIG. 2.—FLUCTUATIONS OF WATER LEVEL IN THE WILLIAMS WELL, SAN BERNARDINO AREA, SANTA ANA RIVER BASIN (CALIFORNIA) 1892–1940

the infrequent periods of excessive precipitation. The Williams well (Fig. 2) is near the Santa Ana River, northwest of Redlands, Calif., and penetrates alluvium in the San Bernardino Valley (10). The observations were made by the Gage Canal Company.

Following a period of deficient precipitation in the late nineties, the water level in the Williams well—the fluctuations in which are reasonably typical of those in wells in the San Bernardino Valley—declined to a low stage in 1904, and then rose persistently until 1917. From 1917 to 1937 the trend was predominantly downward, although small rises occurred in 1922 and 1923. Large rises in water level occurred in 1937 and 1938 as the result of increased precipitation and recharge, after which the water level resumed a downward trend. In 1941 the water level rose again, reaching a stage only about 3 ft lower than that in 1917. The trend was again downward in 1942, but in 1943 the water level rose within 2 ft of the 1941 stage. On the average, the period beginning with 1937 has been a period of relatively high recharge, as was the period from 1904 to 1917.

In areas with a more humid climate the conditions are somewhat different. Recharge generally occurs each year, and only in exceptionally dry years does little or no recharge occur. Fig. 3 shows fluctuations of water level in a shallow

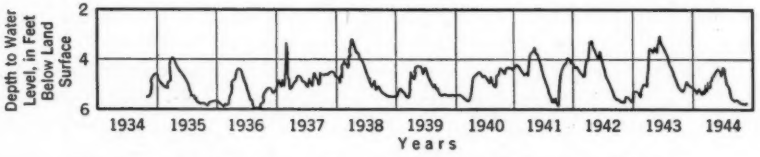

FIG. 3.—HYDROGRAPH OF THE ROSCOMMON WELL IN MICHIGAN, 1934–1944

observation well penetrating glacial outwash sand near Roscommon, in the north-central part of the lower peninsula of Michigan. In the vicinity of the well the ground-water supply is recharged by melting snow and rainfall, and is discharged by evaporation, by transpiration by young trees, and by seepage into a stream several hundred feet away. The water level is not affected by pumping. Substantial recharge has occurred during each year of record, although the amount varies from year to year. Fig. 3 shows no net decline in water level during the period of record. The relatively high stages in 1938, 1941, 1942, and 1943 were caused by excessive or favorably distributed precipitation. The relatively low stage in 1944 was the result of unfavorably distributed precipitation rather than lack of precipitation. The total precipitation in 1944 through October, at the precipitation station nearest the Roscommon well, was about average; but it was unfavorably distributed for ground-water recharge. Precipitation was light during the winter of 1943–1944 —which accounts for the relatively small spring rise in water level. Rainfall was considerably above normal in June, 1944; but, because of evaporation and transpiration, only a small rise in water level resulted.

The abrupt rise and decline in water level in the Roscommon well in February, 1937 (shown in detail in Fig. 4), was obviously exceptional. It was caused mostly, if not entirely, by local recharge from melting snow. Water from melting snow was observed to collect in a surface depression near the well. Although recharge undoubtedly occurred throughout the region as a result of

the thaw, the recharge in the surrounding area was not as great as that at the well, and the water was able to drain away quickly into the adjacent areas.

In some places large and sudden rises, followed by rapid declines, in water-table wells have been attributed to pressure effects produced when the soil becomes saturated; but the data do not show whether such a mechanism may have accounted for part of the February, 1937, rise in the Roscommon well.

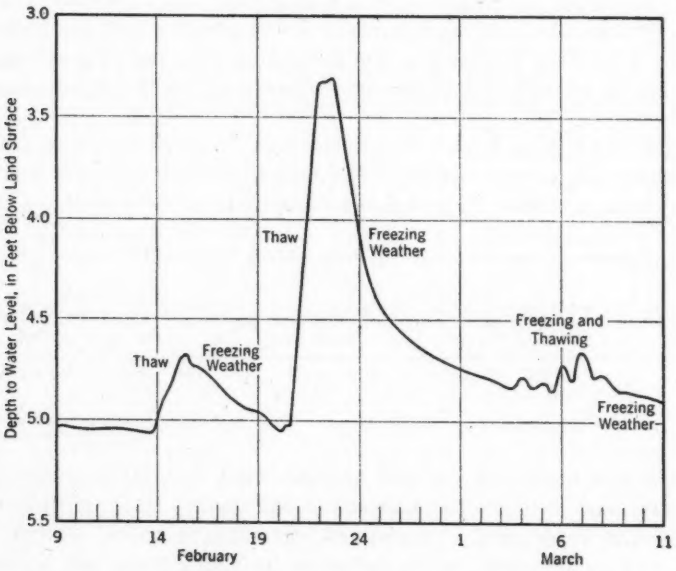

FIG. 4.—ENLARGEMENT OF HYDROGRAPH, FEBRUARY AND MARCH, 1937; THE ROSCOMMON WELL IN MICHIGAN

The relation of forests and the cultivation of land to ground-water supplies has received much discussion, but further rigorous investigation of the subject is needed. It seems inevitable that changes in the land such as deforestation and crop production should have produced changes in the average annual infiltration, changes in the average annual loss by evaporation and transpiration, and hence changes in the average stage of the water table and average annual amount of ground-water runoff. There is no reason to believe, however, that such effects have been progressive over long periods in most parts of the United States.

Drainage ditches, of course, accomplish their purpose of lowering the water table in the areas drained, with a resultant loss in ground-water storage. However, this effect may be expected to be complete within a relatively short time after the drains are installed (a year or a few years at the most) so that a progressive decline of the water table over a long period is not to be expected as the result of the installation of a given system of drains.

Effects of Withdrawal of Water from Wells

Although much investigation will be necessary to establish fully the effects on ground-water supplies of such activities as deforestation, land cultivation, and drainage, the most pressing problems are those created by withdrawal of large quantities of water through wells.

Water-bearing formations differ greatly in their capacity to transmit water and in the quantity of water that they yield from storage with a decline in head. Formations of low permeability that are capable of yielding only small supplies of water for domestic and stock use are found in most parts of the United States. Formations that are capable of yielding sufficient water for large public supplies, irrigation, extensive industrial use, or other purposes requiring large quantities of water are of more limited extent. In their capacity to transmit and store water, these productive aquifers differ widely. Some are of low permeability but have high artesian heads or are buried deeply so that large drawdowns in wells are practicable, and thus the aquifers can be made to yield substantial quantities of water to wells—at least for a limited period. Other aquifers are quite permeable but have low rates of recharge because of lack of precipitation or because of limited outcrop areas. Still others are highly permeable and have high rates of recharge. For the purpose of this paper, the aquifers from which large quantities of water are withdrawn can be divided into two very broad classes: (a) Those with high rates of recharge, transmission, and discharge, which are therefore capable of furnishing large perennial supplies; and (b) those with low rates of recharge and small perennial supplies, where heavy withdrawals are largely from storage.

The effects of withdrawal of water from wells are aggravated in some places by droughts, or by encroachment of salty water. The effect of droughts is most evident on aquifers with small storage capacities, which depend on sustained recharge to support heavy withdrawals. Aquifers with low rates of recharge and small perennial supplies, where heavy withdrawals are largely from storage, are relatively unaffected by droughts.

Encroachment of salty water affects aquifers of both classes but especially those of the second class. It occurs when the head of the fresh water is lowered by withdrawal of water through wells sufficiently to permit the salty water to move toward the wells. Some aquifers in the interior of the United States are subject to encroachment of water of poor quality; but most aquifers in danger of salt-water encroachment are near the coast, where water from the sea, or salty water from part of an aquifer that has not been flushed out by fresh water, tends to move toward the wells as the water levels are lowered by pumping or natural flow. In areas threatened by salt-water encroachment, the best method of detecting the approach of salty water is by means of so-called outpost wells, and the only method of guaranteeing a perennial supply of uncontaminated water is to permit the establishment of a ground-water divide between the salty water and the wells, by reduction or redistribution of pumpage or artificial recharge.. In places where contamination has already occurred, the value of the ground-water supply can be salvaged to a considerable extent by using the water for purposes in which quality is not so important, such as cooling.

Examples of aquifers with high rates of recharge and discharge are the productive Tamiami limestone in the Miami (Fla.) area, the Ocala limestone of central and northern Florida and coastal Georgia, and some deposits of glacial outwash sand and gravel in the northern states—for example, (a) those in the valleys of the Ohio River and several of its northern tributaries; and (b) some highly productive glacial gravels in the Pacific Northwest. Aquifers with low rates of recharge (where heavy withdrawals are largely from storage) include the Ogallala formation in parts of the High Plains (as in the Panhandle of Texas), the Dakota sandstone of the Dakotas and adjacent states, and some of the desert basins or bolsons of the West—for example, the Mimbres Valley in New Mexico. Several representative aquifers of both classes will be described to show the great variety of ground-water conditions in the United States.

Examples of Aquifers with High Rates of Recharge and Large Perennial Supplies

The water-bearing limestone in the Miami area is one of the most permeable and productive aquifers in the United States (11). The rate of recharge from precipitation is high, and of course so is the rate of natural discharge through seeps and springs and by evaporation and transpiration. The coefficient of transmissibility (expressed as the number of gallons of water per day that will move through a section of the aquifer 1 mile wide under a hydraulic gradient of 1 ft per mile) has a value of 2,000,000 to 3,000,000 in the vicinity of Miami, whereas in many heavily pumped aquifers in the United States the value is less than 50,000.

In some places the water supply of the productive aquifer in the Miami area is jeopardized by salt water, which has access to certain areas of heavy pumping, especially by way of drainage canals. The well field of the Miami public water supply was actually affected by salt water that gained access through a canal, and it has been necessary to install a movable dam in the canal to prevent salt water from reaching the well field during periods of low flow in the canal. Also, a "tongue" of salty water is moving slowly toward the well field underground along the canal, and eventually will reach the well field unless a certain minimum fresh-water head, established by redistribution of pumpage or by some other method, is maintained continuously in the area between the salt-water tongue and the well field (11). However, the investigations of the U. S. Geological Survey have shown that, with proper management, the cities of Miami, Miami Beach, and Coral Gables (in Florida) have a very large perennial supply of fresh water.

Areas of productive glacial outwash deposits include those in the Spokane Valley-Rathdrum Prairie area, in Washington and Idaho. The glacial outwash deposits, at least 500 ft thick in many places and very permeable, serve as the underground outlet for water from an arm of Lake Pend Oreille, and the underflow is estimated to be not less than 1,000 cu ft per sec, or about two thirds of a billion gallons per day (unpublished reports by A. M. Piper and L. C. Huff in the files of the U. S. Geological Survey). An average of about 100 cu ft per sec, or about 65 mgd is withdrawn for use in the Spokane Valley.

Glacial outwash deposits in the valleys of the Ohio River and some of its tributaries yield large supplies of water to wells at many places. There is much recharge from rainfall and melting snow, but the river forms a major source of recharge at some places where wells have been constructed near the river and are pumped heavily enough to lower the water table adjacent to the stream. Examples of such supplies (unpublished reports in the files of the U. S. Geological Survey) include the public water supply at Parkersburg, W. Va., and the water supplies at large war plants at Point Pleasant, W. Va., and Charlestown, Ind. At one war plant as much as 50 mgd has been pumped from seven wells in a 2-mile stretch along the banks of the Ohio River. Similar conditions exist in many other valleys. Many large industrial and municipal water supplies have been developed from wells that are so situated as to induce recharge from the streams, and many others undoubtedly will be developed in the future.

In the Mill Creek Valley Water Supply Project, installed by the Federal Works Agency, water is pumped from wells penetrating permeable glacial outwash sands and gravels in the Miami River Valley south of Hamilton, Ohio, and is piped to a large plant in the Mill Creek Valley to the southeast. The designed capacity of the system is 15 mgd. The wells are located so as to take advantage of recharge from precipitation in the immediate vicinity and to intercept ground water flowing westward from a ground-water divide at the head of the Mill Creek Valley; but considerable water will be recharged from the Miami River when wells near the river are pumped heavily (12).

In the Louisville (Ky.) area, large quantities of water are pumped from wells, mostly from wells penetrating glacial outwash deposits in the Ohio Valley. The average pumpage increased from about 37 mgd in 1937 and 1938 to about 62 mgd in 1943 (13). A large part of the total recharge, which is estimated to be from 30 mgd to 40 mgd, is believed to come from the Ohio River.

The pumpage at Louisville has been in excess of recharge for some years, and water has been withdrawn from storage with resulting declines in water levels. In 1943 from 20 mgd to 30 mgd was withdrawn from storage. Inasmuch as the outwash deposits are only about 100 ft thick, the storage has been greatly depleted, and substantial reduction in the rate of pumping will be necessary unless the supply can be greatly increased by wells near the river or by artificial recharge.

In the Platte River Valley of Nebraska, which is cut into deposits of sand, gravel, silt, and clay of glacial and fluviatile origin, recharge of the groundwater supply is largely from precipitation. There is some recharge from the river, however, when increased flows occur following the dry season and also during floods, whenever they occur (14)(15). During most of the year, however, the water table is above stream level and water moves underground toward the river rather than away from it. The contours of the water table show that water recharged from precipitation moves toward the river and discharges into it. However, the movement has a downstream component because of the slope of the valley, and thus the individual particles of water have an oblique path with respect to the river. The downstream component can be considered underflow; but it is small in comparison with the total amount of water re-

charged and discharged and can scarcely be compared with true underflow such as that in the Spokane Valley.

An example of productive water-bearing formations in which droughts may aggravate the effects of heavy pumping is the Indianapolis (Ind.) area. The principal aquifer consists of glacial outwash deposits, but these deposits overlie and furnish water to limestones from which a large quantity of water is also pumped. The effects of two successive years of drought, 1940 and 1941, combined with the effects of record pumping in 1941 to produce critical ground-water conditions in some parts of the area (16). The average pumpage in 1941 was about 52 mgd. During 1942, however, the precipitation was about normal, the pumpage of ground water decreased to about 47 mgd, and the over-all ground-water conditions improved considerably (16).

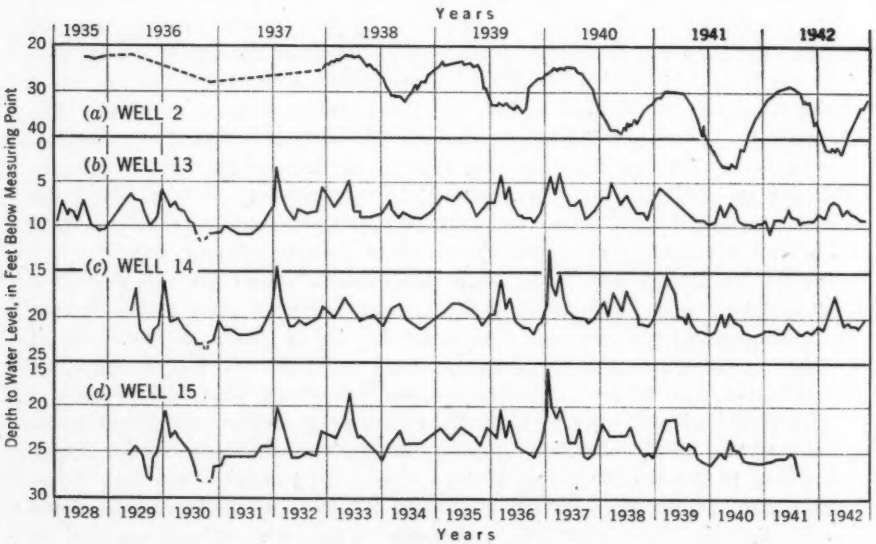

FIG. 5.—HYDROGRAPHS OF SELECTED WELLS, 1928–1942; INDIANAPOLIS, IND.

Wells 13, 14, and 15, Fig. 5, show fluctuations of the water table in an area where recharge is high and where pumping from wells is fairly heavy. The wells penetrate glacial outwash sand and gravel. A total of several million gallons per day is pumped from a gravel pit and from a number of drilled wells. The water levels in the wells were low in 1940 and 1941 as the result of drought and pumping. However, they were not as low as during the drought of 1930–1931. The water levels recovered considerably in 1942, but were much lower than in years of large recharge, such as 1932 and 1937. It is apparent that, because of a high average rate of recharge, the effects of pumping in this area are relatively slight except in years of drought.

Fluctuations in an area where the rate of recharge is relatively low are shown in Fig. 5(*a*). Well 2 penetrates glacial outwash sand and gravel in the

downtown district of Indianapolis, where percolation of water from above is reduced or prevented by pavements and buildings and recharge largely results from lateral percolation from surrounding areas. As the result of heavy pumping, the water levels in wells showed net declines for several years and reached a low stage in 1941, when recharge was reduced by drought. In the summer of 1941 the pumping water levels in some of the wells (about 100 ft deep) penetrating sand and gravel in the downtown district were not far above the bottoms of the wells. As a result of increased recharge in 1942 the water levels rose somewhat but, because of continued heavy pumping, were considerably lower than in 1940 and previous years.

It is estimated that, since pumping began in the Indianapolis area, a total of 400,000,000,000 gal to 500,000,000,000 gal of water has been pumped from wells. Of this volume, about 97% is estimated to have come from recharge and only about 3% from storage. It is because of the dependence on a high rate of recharge to support the heavy pumping that the supply is affected so seriously by prolonged droughts.

These examples illustrate aquifers in which water-table conditions exist at most places. An example of an artesian aquifer with a high rate of recharge and discharge in the Ocala limestone of Florida and Georgia (9)(17)(18). This aquifer is recharged principally in its outcrop area in Georgia, Alabama, and South Carolina, and in two areas in northern and central Florida where recharge occurs through lakes and sinkholes. The water is discharged through submarine springs along the Atlantic and Gulf coasts and through large springs in north-central Florida. Silver Springs and Blue Springs and most of the other large springs in the Florida peninsula are derived from the Ocala limestone and associated limestones. Water supplies of many millions of gallons per day are derived from wells penetrating the Ocala limestone and associated limestones in Florida and Georgia, and are used for public water supply, irrigation, and industrial purposes.

The aquifer is not everywhere equally permeable. It is most permeable and productive in and near the recharge areas, and shows a general decrease in permeability away from the recharge areas. However, there are substantial differences in permeability in areas at approximately equal distances from the recharge areas. For example, at Brunswick, Ga., and Jacksonville, Fla., the Ocala limestone is relatively productive, but at Fernandina, Fla., between Jacksonville and Brunswick, the aquifer is somewhat less productive. In 1943, the draft at the three places ranged from 34 mgd to 40 mgd, as shown in Fig. 6(*a*), which also shows the pumpage at Savannah, Ga. In Fig. 6, the contours (interval, 10 ft) represent approximately the height to which water would rise above mean sea level in tightly cased wells penetrating the principal artesian aquifer in 1943 (Fig. 6(*a*)) and in the first wells ever drilled into the Ocala limestone (Fig. 6(*b*)). The piezometric surface was still above the land surface at Jacksonville and Brunswick, except in the immediate vicinity of pumped wells, whereas, at Fernandina, the piezometric surface was below the land surface in the heavily pumped area. The contrast between conditions at Fernandina and those at Brunswick and Jacksonville is shown strikingly in Fig. 6(*a*). The pumping at Brunswick has caused only a slight lowering of the

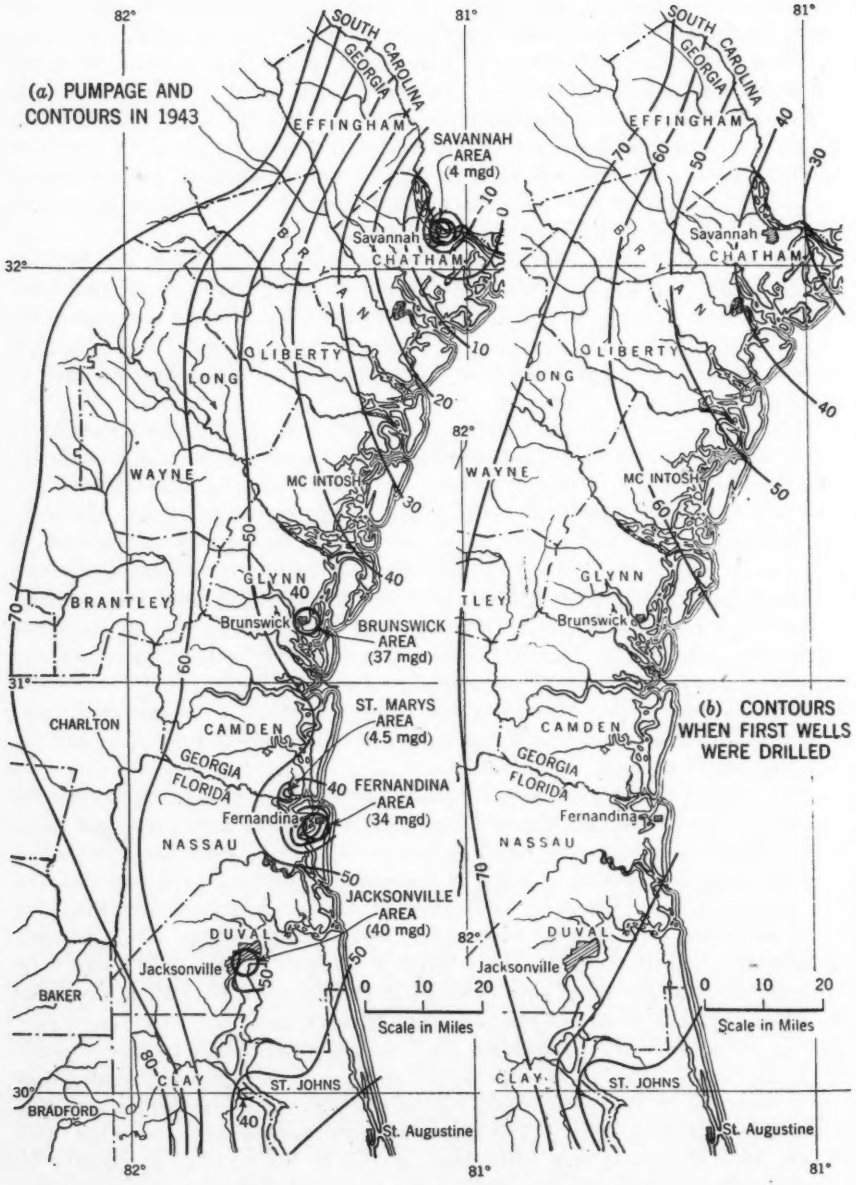

FIG. 6.—PIEZOMETRIC SURFACE OF THE OCALA LIMESTONE IN THE COASTAL AREA OF GEORGIA AND NORTHEASTERN FLORIDA

piezometric surface, whereas at Fernandina it has created a substantial cone of depression. The pumping at Jacksonville has created a cone of depression slightly deeper than that at Brunswick, but considerably shallower than that at Fernandina. A substantial cone of depression has formed at Savannah, where the productiveness of the aquifer is roughly comparable to that at Fernandina.

Where problems of depletion by pumping, not aggravated by drought, have developed in productive aquifers, the cause has generally been concentrated pumping from closely-spaced wells. In many such areas the remedy consists in wider spacing of wells and redistribution of pumpage. In the Baton Rouge (La.) area, as much as 50 mgd has been pumped from the so-called 400-ft and 600-ft sands in an area of about 6 sq miles—much of it in an area of not much more than 1 sq mile (unpublished reports in the files of the U. S. Geological Survey by J. C. Maher and G. C. Taylor, Jr.). Although the water-bearing sands are very productive, the concentrated pumpage in a small area has lowered the water levels in the wells to such an extent that it has been necessary to adopt emergency conservation measures at some of the important industrial plants to insure continued production. Among these measures is the development of supplementary water supplies from the Mississippi River. The ground-water problem in the Baton Rouge area can be solved by distributing the wells that penetrate the 400-ft and 600-ft sands over a larger area and by developing additional water from other sources, including deeper sands in the heavily pumped area and the virtually untouched deposits of alluvium adjacent to the Mississippi River.

Examples of Aquifers with Relatively Small Perennial Supplies

An example of a less productive aquifer which has a fairly high potential rate of recharge but which is strongly affected by drought is the shallow aquifer in the Elizabeth City (N.C.) area (19). The aquifer consists of surficial sands of Pleistocene age, about 30 ft thick. The climate is humid, the average rate of recharge is fairly high, and a moderately large quantity of water can be developed from a system consisting of numerous wells with small individual yields distributed over a large area. A supply of this kind has been developed for Elizabeth City. Because of the small saturated thickness of the aquifer, the successful development of a large supply depends on sustained recharge from precipitation. During prolonged droughts, the small amount of storage is quickly depleted, but with the return of wet weather the available storage space is refilled.

Among the aquifers with a low average rate of recharge, where heavy withdrawals are largely from storage, is the Ogallala formation, the principal aquifer underlying the High Plains. The formation consists largely of a sheet of alluvium deposited in Tertiary time, overlying older dissected rocks. It ranges in thickness from a featheredge at the border of the plains to more than 500 ft in some places. The formation, especially the lower part, consists largely of sand and gravel and contains an enormous amount of water in storage.

Investigations of the hydrology of the Ogallala formation by the U. S. Geological Survey and cooperating agencies are under way in most of the area

underlain by the formation (20)(21)(22)(23)(24)(25)(26)(27)(28)(29). In general, they show that the formation is recharged by precipitation but that the rate of recharge is low. In different places the annal recharge from precipitation probably ranges from a fraction of an inch to a few inches of water.

A report by W. L. Broadhurst and W. H. Alexander, Jr. (20), in 1944, shows the situation that exists in the High Plains in Texas. From 1935 to the end of 1943 the number of irrigation wells in this region increased from about 300 to about 2,950, and the area irrigated increased from 35,000 acres to 400,000 acres, or about 13% of the total area within the boundaries of the irrigated districts. From 1937 to 1941, a period of subnormal precipitation, the increase in irrigation was accompanied by a persistent decline of the water table (Fig. 7). During 1941 the precipitation was heavy, the greatest on record in parts of the region. Consequently, the requirements for irrigation were small, recharge to the underground reservoirs was unusually large, and observation wells in each pumping district showed a pronounced rise in water level. During 1942 the precipitation was again above average, pumping was again light, and the water levels in most of the wells either remained nearly stationary or rose slightly. During 1943 the precipitation was light, the pumping was greater than ever before, and practically all the wells showed a decline in water level (Fig. 7).

It should be stated that the rainfall in 1941 was exceptional and the pumping for irrigation in both 1941 and 1942 was light. The recharge that occurred in 1941 was probably greater than in any year since 1915 and several times the recharge in most years (21). Nevertheless, net declines in water level occurred in the Plainview and Hereford districts; and, although net rises occurred in the Lubbock-Littlefield and Muleshoe districts, the water levels generally resumed their downward trend in 1943. It is evident that the rate of pumping will eventually have to be curtailed greatly in this region.

The Dakota sandstone (which underlies most of North Dakota and South Dakota and considerable parts of near-by states, extending to the south as far as New Mexico) is an aquifer in which water is confined under pressure by thick beds of younger rocks, mostly shale (30). The sandstone crops out at relatively high altitudes in the foothills of the Rockies and those of the Black Hills, and is recharged in these areas. The artesian pressure was 200 lb per sq in., or even more, at some places when the first wells were drilled, and the wells had initial flows of several hundred gallons per minute (31)(32)(33)(34)(35). However, the average permeability of the sandstone is rather low, the specific capacity of the wells—that is, their yield per unit of drawdown—is small, and most of the water represented by the large initial yields was derived from storage by compaction of the beds as the head was lowered.

The high initial yields of the wells gave a false impression of the productiveness of the aquifer and encouraged the drilling of many wells and the withdrawal of excessive quantities of water throughout the Dakota artesian basin. However, as the artesian head was lowered, the yield of the wells decreased; and in some areas the yield is slowly approaching the perennial rate of recharge. For example, in one row of townships in the Ellendale-Jamestown area of North Dakota, the total yield of flowing wells penetrating the Dakota sandstone was about 520 gal per min in 1938. The estimated perennial recharge is about 500

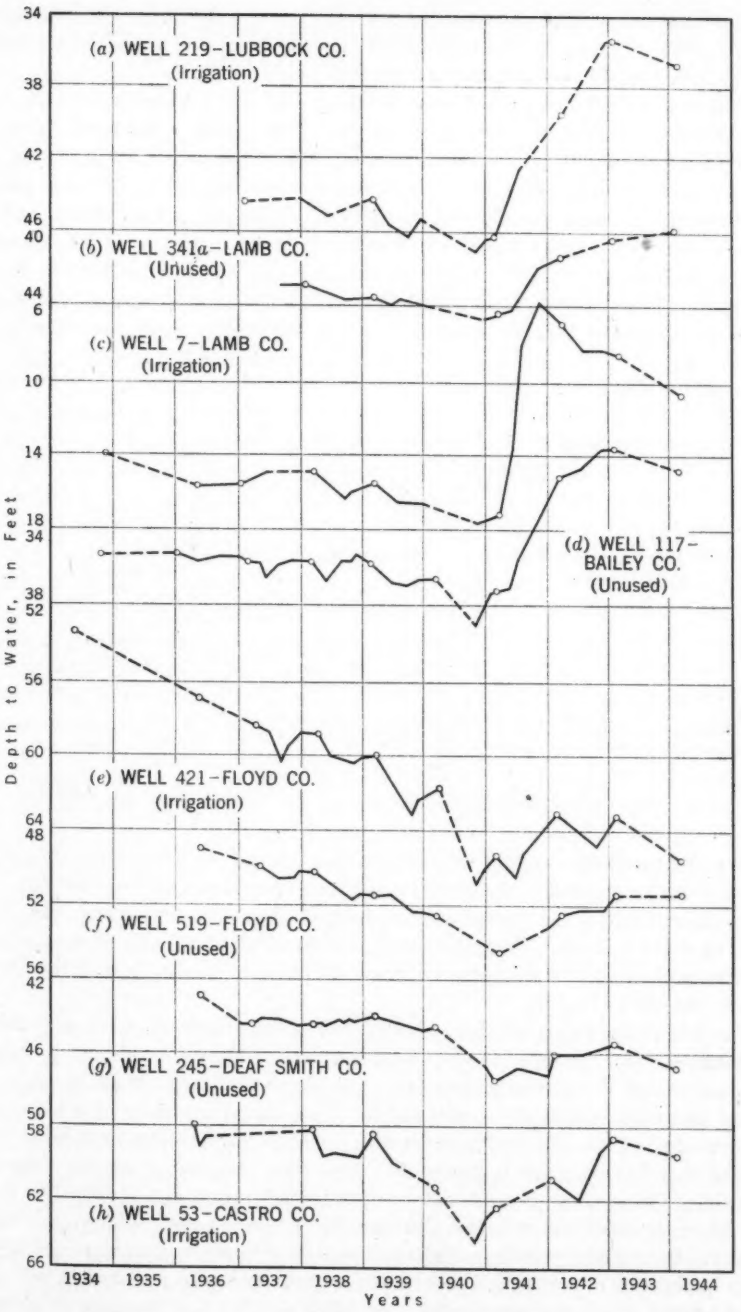

FIG. 7.—HYDROGRAPHS OF WELLS IN THE HIGH PLAINS OF TEXAS

gal per min (30). The yield of the wells in this row of townships in 1923 was about 1,000 gal per min, and the total yield when the wells were being drilled, largely between 1900 and 1920, was doubtless much greater.

Somewhat analogous conditions are found in the region in Illinois, Iowa, and Wisconsin underlain by Cambrian and Ordovician sandstones, although the artesian head was never as high as that in the Dakota Basin, partly because of a smaller difference in altitude between the outcrop areas and the areas of withdrawal. Comparatively few detailed investigations have been made of these aquifers, but great declines in water level have occurred in large parts of the region and much of the water withdrawn has been derived from storage (36)(37). The head of a well drilled at Chicago, Ill., in 1864 was 80 ft above the surface and about 692 ft above sea level (36). This well penetrated the

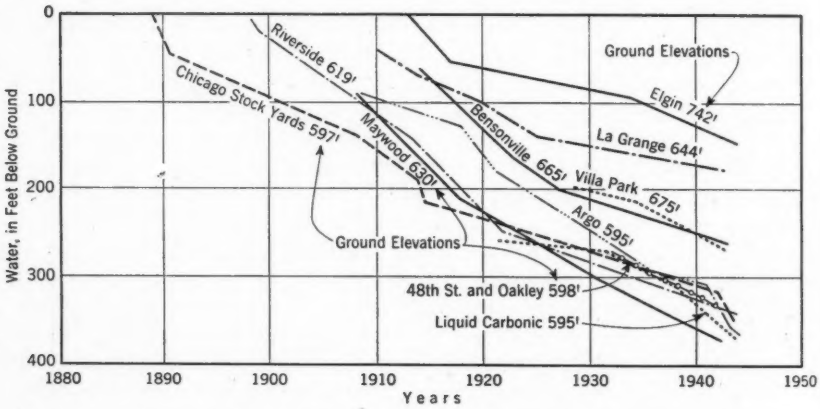

FIG. 8.—DECLINE OF WATER LEVELS IN WELLS IN AND NEAR CHICAGO, ILL., 1899–1943.

Galena-Platteville limestone, which doubtless had a head lower than that of the deeper Cambrian sandstones that have since been developed so heavily. In 1943, according to data issued by the Illinois State Water Survey, the water levels in some wells at Chicago, penetrating the Cambrian sandstones, were at altitudes as low as 230 ft above sea level, and the decline undoubtedly continued during 1944 (Fig. 8).

The Mimbres Valley in New Mexico has a very large quantity of water in subterranean storage and many productive wells. It was assumed to have a large perennial supply until critical investigation showed that the average annual recharge is actually quite small. The valley consists of a structural basin, floored with relatively impervious consolidated rocks and filled with alluvial deposits of gravel, sand, silt, and clay, derived from the adjacent mountains (38). The ground-water supply is recharged largely by the Mimbres River, which sinks into the alluvium after leaving the mountains. Rainfall on the floor of the valley and return flow from irrigation contribute relatively minor quantities of water. The average annual recharge was estimated to be about 10,000 acre-ft for the period from 1908 to 1928—an average of about 9

mgd. Records of water levels in wells, and of stream flow, show that recharge was slight during the period from 1929 to 1940, inclusive. Even in 1941, an exceptionally wet year, the recharge was not very great. Recharge from the Mimbres River is probably less than it was formerly because of the greater use or river water for irrigation in the headwater areas.

The total pumpage, largely for irrigation, increased from about 10,000 acre-ft per yr during the period from 1929 to 1933 to about 25,000 acre-ft in 1940 (an average of about 22 mgd), and decreased only to about 21,000 acre-ft in the exceptionally wet year of 1941. The records of water levels in wells show that most of the water pumped has been derived from storage in the subterranean reservoir. Even in 1941 there was a net loss in storage of several thousand acre-feet in the valley as a whole.

The Mimbres Valley is one of the areas in New Mexico in which legal control over the ground-water supply is exercised by the state. Although the pumpage is in excess of the average rate of recharge, the decline in ground-water levels at the present rate of pumping will be gradual, and the present policy is not to reduce the pumpage but simply to restrict additional developments.

Legal control of the use of ground water is a broad subject that cannot be treated adequately in this paper, but an example of a sound approach may be cited—namely, the New Mexico ground-water law just mentioned in connection with the Mimbres Valley. The law was an outgrowth of the overdevelopment of the artesian water supply of the Roswell basin in New Mexico. In this area the high initial yields of flowing wells penetrating cavernous Permian limestone encouraged the drilling of so many wells and the withdrawal of so much water that the artesian head declined sharply; wells ceased flowing along the margin of the flowing well area, which was becoming smaller and smaller; and large investments in land were threatened. The U. S. Geological Survey, in cooperation with the state engineer, made a thorough investigation of the region, determined the perennial safe yield of the artesian aquifer, and assisted state officials in drafting a law for the control of the ground-water supply. The law was enacted and has been enforced effectively by the state engineer with the cooperation of the ground-water users; thus the valuable artesian water supply is both efficiently developed and secure (39)(40). Conservation of water is effected by preventing wasteful use of water for irrigation and by reducing loss of water from the limestone into the overlying valley fill by locating and plugging leaky wells.

The New Mexico ground-water law is regarded as a model. Its development was made possible by the thorough investigation of the ground-water supply of the aquifer concerned and the determination of the rate at which water could be withdrawn perennially. Unless a good estimate of the perennial water supply of an aquifer has been made available through adequate investigation, a law enacted for control of the water supply of the aquifer is likely to be difficult or impossible to enforce.

Probably the most serious problems of ground-water depletion will arise in the areas underlain by aquifers that have low rates of recharge but from which wells can obtain initial yields of several hundred gallons per minute—or even 1,000 gal per min—giving the impression of perennial supplies more abundant than they actually are, and leading to overoptimistic and excessive develop-

ment. The impression of abundance may be caused by high permeability, as in the Mimbres Valley, or to high artesian head, as in the Dakota artesian basin.

Artificial Recharge

A method of conserving ground-water supplies that will be used more and more in the future is artificial recharge, in which water from streams or water pumped from wells is recharged to the ground-water reservoirs (41). Artificial recharge has been practiced on a large scale for some years in California, where recharge has been accomplished by passing water from streams or other bodies of surface water over the ground—in ditches, in furrows, in basins, or in a thin sheet, by a method commonly known as "water spreading." The infiltration galleries from which the public water supply of Des Moines, Iowa, is obtained are recharged artificially from shallow basins excavated over the galleries. Similar projects are in operation at a few other places in the eastern part of the United States, including Dayton, Ohio. Recharge from an artificial lake is accomplished in the Duhernal water-supply development near Old Bridge, N. J.

A method that has been used on a large scale in relatively few places is artificial recharge through wells. The most notable example is on western Long Island in New York, where water pumped for cooling from wells at more than a certain minimum rate must be returned to the ground through wells constructed for the purpose (42)(43). The New York State Water Power and Control Commission has the authority to require similar practices elsewhere on Long Island if necessary.

At Louisville where the present pumpage of ground water is in excess of the recharge and the ground-water levels are declining, artificial recharge of certain wells was undertaken at the suggestion of the U. S. Geological Survey and through the efforts of the War Production Board and the industries concerned. This recharge permitted continued production of industrial alcohol at certain plants whose production otherwise would have had to be curtailed because of lack of water (44). Wells are also being recharged on a fairly large scale at Indianapolis (16), at a large industrial plant in Virginia, and probably at several other places. It is to be expected that, at many places, wells will be recharged with cold filtered surface water in the winter, where the value of ground water for cooling purposes makes this practice economically feasible. This type of recharge is practiced at Louisville and at the industrial plant in Virginia.

Methods of Investigation

The hydrology of ground-water reservoirs is a somewhat complicated subject, and each reservoir has its own peculiar characteristics, due both to natural conditions and to the extent and distribution of the ground-water developments. Thus, each reservoir must be given individual study. The science of ground-water hydrology has advanced greatly in recent years, and methods are becoming available for the comprehensive study of the major ground-water reservoirs that will show the quantities of water that are perennially available and how to develop and utilize them efficiently.

The U. S. Geological Survey is engaged actively in the development of methods for making pumping tests and analyzing the results with respect to: (*a*) The quantities of ground water that are available in specific areas, (*b*) the

extent of interference between different wells or groups of wells, and (*c*) the additional supplies that can be developed by proper distribution of the pumpage (45)(46)(47). Important developments have been made in recent years on the basis of the Theis nonequilibrium method, which permits the determination, from simple pumping tests, of the coefficients of transmissibility and storage of an aquifer—that is, the rate at which water is transmitted under a given gradient and the quantity of water released from storage with a given lowering in head. These two fundamental coefficients are then utilized to predict the effects of pumping given installations of wells penetrating the aquifer. Refined methods are coming into use for analyzing the results of pumping tests in aquifers that are limited by relatively impermeable boundaries within a short distance from the wells or in aquifers whose sources of recharge are near by. Under favorable conditions the results of pumping tests can even be used to determine the presence of geologic structures, such as a fault or other barrier that may not be apparent from surface observation or well records.

Conclusion

Deforestation, cultivation of land, drainage, and similar practices have undoubtedly affected ground-water supplies to some extent in the areas involved, but there is no reason to believe that the effects have been progressive over long periods of years in most parts of the United States. The effects of pumping or natural flow from wells are of great concern; and, as has been shown by the numerous examples of aquifers of different kinds that have been described, the conservation and development of ground-water supplies in the future will require thorough investigation of each major aquifer and effective legal control of the withdrawals from the more heavily developed aquifers. In some places heavy withdrawals from wells are depleting the ground-water supply and cannot be continued indefinitely; in many other places recharge is large and therefore large supplies will be perennially available to wells.

APPENDIX.—BIBLIOGRAPHY

(1) **"Symposium on Fluctuations of Ground-Water Level,"** *Transactions*, Am. Geophysical Union, Vol. 17, 1936, Pt. II, p. 337. (*a*) "Review of the Work of W. J. McGee on Ground-Water Levels," by O. E. Meinzer, pp. 386–390. (*b*) "Fluctuations in Ground-Water at Woodgate, N. Y.," by E. S. Cullings, p. 359, Fig. 2.

(2) **"Our Water Supply,"** by O. E. Meinzer, *Journal*, Washington Academy of Sciences, Vol. 27, No. 3, March 15, 1937.

(3) **"Ground Water in the United States, a Summary,"** by O. E. Meinzer, *Water-Supply Paper No. 836-D*, U. S. G. S., Govt. Printing Office, Washington, D. C., 1939.

(4) **"Ground Water, a Vital National Resource,"** by O. E. Meinzer, *Journal*, A. W. W. A., November, 1942, pp. 1595–1606.

(5) **"Protection of Ground-Water Supplies,"** by O. E. Meinzer, *Johnson National Drillers Journal*, September–October, 1944, pp. 10–11 and 14.

(6) **"Occurrence, Origin, and Discharge of Ground Water,"** by O. E. Meinzer, Physics of the Earth Series, Vol. 9, "Hydrology," McGraw-Hill Book Co., Inc., New York, N. Y., 1942, Chapter 10-A.

(7) **"Movement of Ground Water and Its Relation to Head, Permeability, and Storage,"** by O. E. Meinzer and L. K. Wenzel, *ibid.*, Chapter 10-B.

(8) **"The Source of Water Derived from Wells,"** by C. V. Theis, *Civil Engineering*, May, 1940. pp. 277–280.

(9) **"Ground-Water Investigations in Florida, with Special Reference to Duval and Nassau Counties,"** by H. H. Cooper, Jr., *Journal*, A. W. W. A February, 1944, pp. 169–185.

(10) **"Water Levels and Artesian Pressure in Observation Wells in the United States in 1940,"** by F. C. Ebert, *Water-Supply Paper No. 911*, U. S. G. S., Govt. Printing Office, Washington, D. C., 1941, Pt. 6, Section on California.

(11) **"Interim Report on the Investigation of Water Resources in Southeastern Florida, with Special Reference to the Miami Area in Dade County,"** by G. G. Parker, G. E. Ferguson, and S. K. Love, Florida Geological Survey (in cooperation with U. S. G. S., Dade County, and the cities of Miami, Miami Beach, and Coral Gables), June, 1944 (mimeographed).

(12) **"A Quantitative Study of the Well Fields of the Mill Creek Valley Water Supply Project, Butler County, Ohio,"** by F. H. Klaer, Jr., and R. G. Kazmann, U. S. G. S. (in cooperation with Federal Works Agency), July, 1943 (hectographed).

(13) **"Progress Report on the Ground-Water Resources of the Louisville Area, Kentucky,"** by W. F. Guyton, W. T. Stuart, and G. B. Maxey, U. S. G. S. (in cooperation with Kentucky Dept. of Mines and Minerals and the City of Louisville), March, 1944 (mimeographed).

(14) **"Geology and Ground-Water Resources of South-Central Nebraska, with Special Reference to the Platte River Valley Between Chapman and Gothenburg,"** by A. L. Lugn and L. K. Wenzel, *Water-Supply Paper No. 779*, U. S. G. S. (in cooperation with Conservation and Survey Div., Univ. of Nebraska, Lincoln), Govt. Printing Office, Washington, D. C., 1938.

(15) **"Ground Water in Keith County, Nebraska,"** by L. K. Wenzel and H. A. Waite, *Water-Supply Paper No. 848*, U. S. G. S. (in cooperation with Conservation and Survey Div., Univ. of Nebraska, Lincoln), Govt. Printing Office, Washington, D. C., 1941.

(16) **"Ground-Water Resources of the Indianapolis Area, Marion County, Indiana,"** by C. L. McGuinness, Indiana Div. of Geology (in cooperation, with U. S. G. S.), January 1, 1943 (mimeographed).

(17) **"Artesian Water in the Florida Peninsula,"** by V. T. Stringfield, *Water-Supply Paper No. 773-C*, U. S. G. S. (in cooperation with Florida Geological Survey), Govt. Printing Office, Washington, D. C., 1936.

(18) **"Artesian Water in the Coastal Area of Georgia and Northeastern Florida,"** by V. T. Stringfield, M. A. Warren, and H. H. Cooper, Jr., *Economic Geology*, November, 1941, pp. 698–711.

(19) **"Geology and Ground-Water Resources of the Elizabeth City Area, North Carolina,"** by S. W. Lohman, *Water-Supply Paper No. 773-A*, U. S. G. S. (in cooperation with North Carolina Dept. of Conservation and Development), 1936.

(20) **"Progress Report on Ground Water in the High Plains in Texas,"** by W. L. Broadhurst and W. H. Alexander, Jr., Texas State Board of Water Engineers (in cooperation with U. S. G. S.), 1944 (mimeographed), pp. 11–12.

(21) **"Water Levels and Artesian Pressure in Observation Wells in the United States in 1942,"** by R. W. Sundstrom, *Water-Supply Paper No. 947*, Govt. Printing Office, Washington, D. C., 1944, Pt. 4, Section on Texas.

(22) **"Geology and Ground-Water Resources of Cimarron County, Oklahoma,"** by S. L. Schoff, *Bulletin No. 64*, Oklahoma Geological Survey (in cooperation with U. S. G. S.), 1943.

(23) **"Geology and Ground-Water Resources of Texas County, Oklahoma,"** by S. L. Schoff, *Bulletin No. 59*, Oklahoma Geological Survey (in cooperation with U. S. G. S.), 1939.

(24) **"Geology and Ground-Water Resources of Hamilton and Kearny Counties, Kansas,"** by T. G. McLaughlin, *Bulletin No. 49*, Kansas State Geological Survey (in cooperation with U. S. G. S. and Kansas State Boards of Health and Agriculture), December, 1943.

(25) **"Geology and Ground-Water Resources of Meade County, Kansas,"** by J. C. Frye, *Bulletin No. 45*, Kansas State Geological Survey (in cooperation with U. S. G. S. and Kansas State Boards of Health and Agriculture), 1942.

(26) **"Geology and Ground-Water Resources of Stanton County, Kansas,"** by B. F. Latta, *Bulletin No. 37*, Kansas State Geological Survey (in cooperation with U. S. G. S. and Kansas State Boards of Health and Agriculture), 1941.

(27) **"Geology and Ground-Water Resources of Morton County, Kansas,"** by T. G. McLaughlin, *Bulletin No. 40*, Kansas State Geological Survey (in cooperation with U. S. G. S. and Kansas State Boards of Health and Agriculture), 1941.

(28) **"Geology and Ground-Water Resources of Ford County, Kansas,"** by H. A. Waite, *Bulletin No. 43*, Kansas State Geological Survey (in cooperation with U. S. G. S. and Kansas State Boards of Health and Agriculture), 1942.

(29) **"Geology and Ground-Water Resources of Box Butte County, Nebraska,"** by R. C. Cady and O. J. Scherer, U. S. G. S. (in cooperation with Conservation and Survey Div., Univ. of Nebraska, Lincoln, in course of publication as a *Water-Supply Paper*).

(30) **"Water Supply of the Dakota Sandstone in the Ellendale-Jamestown Area, North Dakota, with Reference to Changes Between 1923 and 1938,"** by L. K. Wenzel and H. H. Sand, *Water-Supply Paper No. 889-A*, U. S.G. S. (in cooperation with North Dakota Geological Survey), Govt. Printing Office, Washington, D. C., 1942.

(31) **"Artesian Waters in the Vicinity of the Black Hills,"** by N. H. Darton, *Water-Supply Paper No. 227*, U. S. G. S., Govt. Printing Office, Washington, D. C., 1918.

(32) **"The Artesian-Water Supply of the Dakota Sandstone in North Dakota, with Special Reference to the Edgeley Quadrangle,"** by O. E. Meinzer

and Herbert A. Hard, *Water-Supply Paper No. 520-E*, U. S. G. S., Govt. Printing Office, Washington, D. C., 1925.

(33) **"Preliminary Report on the Geology and Underground Water Resources of the Central Great Plains,"** by N. H. Darton, *Professional Paper No. 32*, U. S. G. S., Govt. Printing Office, Washington, D. C., 1905.

(34) **Geology and Water Resources of the Northern Portion of the Black Hills and Adjoining Regions in South Dakota and Wyoming,"** by N. H. Darton, *Professional Paper No. 65*, U. S. G. S., Govt. Printing Office, Washington, D. C.

(35) **"Preliminary Description of the Geology and Water Resources of the Southern Half of the Black Hills and Adjoining Regions in South Dakota and Wyoming,"** by N. H. Darton, *Twenty-First Annual Report*, U. S. G. S., Govt. Printing Office, Washington, D. C., Pt. IV(b), 1901.

(36) **"The Artesian Waters of Northeastern Illinois,"** by C. B. Anderson, *Bulletin No. 34*, Illinois State Geological Survey, 1919.

(37) **"Ground Water Supplies of the Chicago-Joliet-Chicago Heights Area,"** *Bulletin No. 35*, Illinois State Water Survey, 1943.

(38) **"Progress Report on the Ground-Water Supply of Mimbres Valley, New Mexico, 1938-41,"** by C. S. Conover and P. D. Akin, State Engr. of New Nexico (in cooperation with U. S. G. S., in course of publication as part of 14th and 15th biennial repts. of the State Engr.).

(39) **"Geology and Ground-Water Resources of the Roswell Artesian Basin, New Mexico,"** by A. G. Fiedler and S. S. Nye, *Water-Supply Paper No. 639*, U. S. G. S. (in cooperation with State Engr. of New Mexico), Govt. Printing Office, Washington, D. C., 1933.

(40) **"Report of Artesian Well Supervisor,"** State Engr. of New Mexico, 12th and 13th biennial repts., 1934–1938.

(41) **"Artificial Recharge of Ground Water,"** by D. G. Thompson, U. S. G. S., February 10, 1942 (mimeographed circular).

(42) **"Ground-Water for Air Conditioning on Long Island, New York,"** by M. L. Brashears, Jr., and R. M. Leggette, *Transactions*, Am. Geophysical Union, Vol. 19, 1938, Pt. I, pp. 412–418.

(43) **"Ground-Water Temperature on Long Island, New York, as Affected by Recharge of Warm Water,"** by M. L. Brashears, Jr., *Economic Geology*, December, 1941, pp. 811–829.

(44) **"Conservation of Ground Water in the Louisville Area, Kentucky,"** by W. F. Guyton and Hazel E. Sublett, Kentucky Dept. of Mines and Minerals (in cooperation with U. S. G. S.), 1944 (mimeographed).

(45) **"Methods for Determining Permeability of Water-Bearing Materials, with Special Reference to Discharging-Well Methods,"** by L. K. Wenzel, *Water-Supply Paper No. 887*, Govt. Printing Office, Washington, D. C., 1942.

(46) **"Some Field Applications of Water Transmissibility and Storage Coefficients,"** by R. G. Kazmann, *Agricultural Engineering*, August, 1944, pp. 299–300 and 304.

(47) **"Drawdown Test to Determine Effective Radius of Artesian Well,"** by C. E. Jacob, *Proceedings*, ASCE, May, 1946, pp. 629–646.

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PAPERS

STRENGTH OF BEAMS AS DETERMINED BY LATERAL BUCKLING

By Karl de Vries,[1] M. ASCE

Synopsis

All usual specifications for the design of beams provide for reducing the allowable unit stress of the compression flange as the unsupported span length increases, to guard against lateral buckling. Such provisions, for the sake of simplicity, relate this reduction only to the ratio of unsupported beam length to flange width, thus neglecting the obvious influence of (*a*) the horizontal moment of inertia of the compression flange; and (*b*) the restraint afforded by the tension flange, which decreases as the depth of the beam increases.

By an extension and an evaluation of the treatment of this problem given by S. Timoshenko,[2] more nearly rational formulas are developed. Typical formulas are proposed for inclusion in specifications, and additional formulas for special investigations are derived and their application is described.

Professor Timoshenko's basic formulas have been accepted without repeating their derivations. He has also tabulated certain values[3] for beam strengths, which are practically identical with the corresponding values of this paper; but a greater range of beam strengths is given in this paper than is presented by Professor Timoshenko. In referring to the Timoshenko formulas, some repetition is unavoidable. Since Professor Timoshenko has not shown the steps for computing the beam strengths and since they will be of interest for the reader who wishes to check the work done here, they are given in Appendix I with the necessary condensation.

Notation.—The letter symbols in this paper are defined where they first appear, in the text or illustrations, and are assembled alphabetically, for con-

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **February 1, 1947.**

[1] Designer, Bethlehem Steel Co., Fabricated Steel Constr., Bethlehem, Pa.

[2] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, Chapter V, p. 239.

[3] *Ibid.*, pp. 267, 268, and 272, Tables 22, 23, 25, and 26.

venience of reference, in Appendix II. Discussers are requested to adapt their notation to this form.

PART I. SIMPLE BEAMS

Concentrated Load at Center

It is assumed that the ends of the simple beam shown in Fig. 1 can rotate freely with respect to the x-axis and the y-axis, but that rotation with respect to the z-axis is prevented. In calculating the critical value of the load P, Fig. 1, Professor Timoshenko assumes that a small lateral buckling has occurred and, from the differential equations of equilibrium, determines the magnitude of the smallest load required to keep the beam in this slightly buckled form:[4,5]

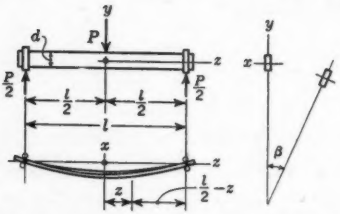

Fig. 1.—Concentrated Load at Center

$$\frac{P^2}{4\,B}\int_0^{l/2}\beta^2\left(\frac{l}{2}-z\right)^2 dz = C\int_0^{l/2}\left(\frac{d\beta}{dz}\right)^2 dz + \frac{B\,d^2}{4}\int_0^{l/2}\left(\frac{d^2\beta}{dz^2}\right)^2 dz \quad \ldots\ldots (1)$$

Eq. 1 expresses the relationship between a concentrated load P applied at the centroid of the beam at midspan; and β, the angular deflection of the beam at any point at a distance z from midspan, in terms of two constants B and C which in turn depend on the properties of the beam. The values of these constants are found as follows:[6] B is the transverse flexural rigidity of the beam, and is equal to $E\,I_y$; C is its torsional rigidity and is equal to $\frac{E\,I_y\,d^2}{4\,a^2}$; E is the modulus of elasticity; I_y is the moment of inertia about the y-axis; and a is a torsional bending constant, values of which, for beams, are listed in manuals.[7]

It is necessary to introduce a correction term in Eq. 1 to allow for the effect of loading not at the centroid but at the top or bottom flange. This correction is $+\frac{P\,d}{4}\beta^2{}_o$ for top flange loading and $-\frac{P\,d}{4}\beta^2{}_o$ for bottom flange loading, in which β_o is the angle of twist of the beam at its center line.

With this modification, substituting the foregoing constants, Eq. 1 yields the basic formula:

$$\frac{P^2}{4\,E\,I_y}\int_0^{l/2}\beta^2\left(\frac{l}{2}-z\right)^2 dz \pm \frac{P\,d}{4}\beta^2{}_o - \frac{E\,I_y}{4}\frac{l^2}{a^2}\frac{d^2}{l^2}\int_0^{l/2}\left(\frac{d\beta}{dz}\right)^2 dz - \frac{E\,I_y}{4}d^2\int_0^{l/2}\left(\frac{d^2\beta}{dz^2}\right)^2 dz = 0 \quad \ldots\ldots (2)$$

[4] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 251, Fig. 146.

[5] *Ibid.*, p. 266, Eq. (*e*).

[6] *Ibid.*, pp. 240–258.

[7] "Bethlehem Manual of Steel Construction," *Catalogue S-47*, Bethlehem Steel Co., Bethlehem, Pa., 1934, p. 285, Table II.

Eq. 2 is a quadratic equation with respect to P, which can readily be solved into:

$$P = \frac{E I_y d}{2} \times \frac{\mp \beta^2{}_o + \sqrt{\beta^4{}_o + 4 \int_0^{l/2} \beta^2 \left(\frac{l}{2} - z\right)^2 dz \left[\frac{l^2}{a^2}\frac{1}{l^2}\int_0^{l/2}\left(\frac{d\beta}{dz}\right)^2 dz + \int_0^{l/2}\left(\frac{d^2\beta}{dz^2}\right)^2 dz\right]}}{\int_0^{l/2} \beta^2 \left(\frac{l}{2} - z\right)^2 dz} \quad (3)$$

The least value of P, as determined by Eq. 3, is the critical load under which beam failure by flange buckling may be expected for these loading conditions. For a concentrated load applied at the top flange, $-\beta^2{}_o$ is used; applied at the bottom flange, $+\beta^2{}_o$ is used; and, applied at the centroid, since the correction factor $\pm \frac{P d}{4}\beta^2{}_o$ disappears, $\beta^2{}_o$ and $\beta^4{}_o$ are omitted. Eq. 3 is not readily soluble; and, to reduce it to a more usable form, the unit stress in the beam for critical load (f) is determined as explained in Appendix I. This unit stress is expressed as:

$$f = \frac{I_y d^2}{I_z a^2} E \frac{a^2}{l^2} k \quad (4)$$

For any beam of given length the values in Eq. 4 are constant, with the exception of the factor k which will vary for different loading conditions. The values of k are as shown in Table 1(*a*).

TABLE 1.—VALUES OF k (SEE EQS. 21, 36, 43, AND 50, RESPECTIVELY) FOR USE IN SOLVING EQ. 4

	Simple Beams						(c) Two-Bay Beams			
$\frac{l}{a}$	(a) Concentrated Load, Eq. 21			(b) Uniform Load, Eq. 36			Concentrated load, Eq. 43	Uniform Load, Eq. 50		
	Top flange	Centroid	Bottom flange	Top flange	Centroid	Bottom flange		Top flange	Centroid	Bottom flange
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2	2.525	3.982	6.272	2.269	3.310	4.824	5.411	3.499	3.917	4.381
3	3.127	4.649	6.885	2.788	3.860	5.341	6.287	4.142	4.560	5.018
4	3.863	5.440	7.632	3.418	4.518	5.969	7.331	4.910	5.329	5.780
5	4.687	6.310	8.469	4.121	5.244	6.669	8.476	5.757	6.174	6.620
6	5.568	7.232	9.365	4.872	6.013	7.417	9.686	6.651	7.069	7.509
7	6.490	8.187	10.30	5.655	6.810	8.197	10.94	7.579	7.995	8.432
8	7.439	9.164	11.26	6.459	7.626	8.999	12.22	8.529	8.944	9.378
9	8.409	10.16	12.24	7.280	8.456	9.817	13.52	9.494	9.909	10.34
10	9.393	11.16	13.24	8.113	9.296	10.65	14.84	10.47	10.89	11.31
11	10.39	12.18	14.24	8.954	10.14	11.49	16.17	11.46	11.87	12.30
12	11.39	13.20	15.26	9.802	11.00	12.33	17.51	12.45	12.86	13.29
13	12.41	14.22	16.28	10.65	11.85	13.18	18.86	13.45	13.86	14.29
14	13.43	15.25	17.30	11.51	12.72	14.04	20.21	14.45	14.86	15.29

UNIFORMLY DISTRIBUTED LOAD

In a manner similar to that described under the heading, "Concentrated Load at Center," values of f, the critical unit stress, for a load uniformly dis-

tributed on the beam, may be determined. The equation for f is identical with Eq. 4, but the values of k for values of l/a from 2 to 14 are not the same as those for a beam with concentrated load (see Table 1(b)).

Critical Stress for Rolled Beams

A comparison of the k-factors in Tables 1(a) and 1(b) indicates that a uniformly distributed load applied at the top flange gives the lowest critical stress; and, since this loading also represents the most usual case, or nearly so, it alone has been used in the following study. Critical stresses for uniformly distributed load applied at the top flange were computed for all "wide-flange" (WF) sections from 36 in. to 10 in. deep, which ordinarily are classified as beams, and also for all "light beams" 12 in. and 10 in. deep—a total of 132 beams.[8] First, the $\frac{I_y d^2}{I_x a^2}$-factors of Eq. 4 were computed for each beam, and then, using the k-values of Table 1, Col. 5, with their corresponding values of $E\frac{a^2}{l^2}$, the critical stresses could be tabulated, omitting, however, any case in which the resulting critical stress would exceed 60,000 lb per sq in. and any case in which l/b (b being the width of the beam flange) would exceed 100. The complete tabulation of all the beams is filed with the Engineering Societies Library in New York, N. Y.[8a] The tabulation for five 14-in. sections is given in Table 2(a), Cols. 2 to 6, and is typical of the work.

In computing the values shown in Table 2, it is assumed that in Eq. 4 the modulus of elasticity $E = 30{,}000{,}000$ lb per sq in.; and that the values of a and $10{,}000\frac{I_y d^2}{I_x a^2}$ are as follows:

14-in. WF	a (in.)	$10{,}000\frac{I_y d^2}{I_x a^2}$
87 lb	104.0	65.54
78 lb	81.86	71.71
61 lb	74.32	58.59
43 lb	69.04	41.27
34 lb	64.28	29.79

For the section 14 WF 43, and a beam length of $5\,a = 5 \times 69.04$ in. $= 345.2$ in., for example, the critical stress is $\frac{41.27}{10{,}000} \times 30{,}000{,}000 \times \frac{1}{25} \times 4.121 = 20{,}410$ lb per sq in.

Entries in Table 2 below the horizontal line represent beams that would be excluded from use under the frequently found arbitrary restriction that l/b shall not exceed 40, yet these beams are found in the present analysis to possess considerable capacity.

To be able to express the critical stresses in terms of readily obtained variables (that is, the properties of the beam sections shown in handbooks), trials of various combinations of the variables that contribute most to the strength of

[8] "Bethlehem Manual of Steel Construction," *Catalogue S-47*, Bethlehem Steel Co., Bethlehem, Pa., 1934, pp. 146 to 158 and p. 167.

[8a] 33 W. 39th St., New York 18, N. Y.

TABLE 2.—CRITICAL STRESS, f, COMPUTED FROM EQ. 4, FOR FIVE 14-IN. WIDE-FLANGE (WF) BEAMS (POUNDS PER SQUARE INCH)

$\frac{l}{a}$	LOAD APPLIED AT THE TOP FLANGE					LOAD APPLIED AT THE CENTROID					LOAD APPLIED AT THE BOTTOM FLANGE				
	87-lb beam	78-lb beam	61-lb beam	43-lb beam	34-lb beam	87-lb beam	78-lb beam	61-lb beam	43-lb beam	34-lb beam	87-lb beam	78-lb beam	61-lb beam	43-lb beam	34-lb beam
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(a) UNIFORMLY DISTRIBUTED LOADS															
2	....	....	....	....	50,700	....	....	....	....	....	....	....	....	....	....
3	....	....	54,440	38,340	27,680	....	....	....	53,100	38,330	....	....	....	....	53,030
4	42,000	45,960	37,550	26,450	19,090	53,530	....	49,640	34,960	25,240	....	....	....	46,190	33,340
5	32,410	35,460	28,970	20,410	14,730	41,240	45,130	36,870	25,970	18,750	52,450	57,380	46,880	33,020	23,840
6	26,610	29,110	23,790	16,760	12,090	32,840	35,930	29,360	20,680	14,930	40,510	44,320	36,210	25,510	18,410
7	22,690	24,830	20,280	14,290	10,310	27,320	29,900	24,430	17,210	12,420	32,890	35,980	29,400	20,710	14,950
8	19,850	21,710	17,740	12,500	9,020	23,430	25,640	20,950	14,750	10,650	27,640	30,250	24,710	17,410	12,570
9	17,670	19,330	15,800	11,130	8,030	20,530	22,460	18,350	12,930	9,330	23,830	26,070	21,300	15,010	10,830
10	15,950	17,450	14,260	10,050	7,250	18,280	20,000	16,340	11,510	8,310	20,930	22,900	18,710	13,180	9,510
11	14,550	15,920	13,010	9,160	....	16,480	18,040	14,740	10,380	....	18,670	20,420	16,690	11,750	....
12	13,380	14,640	11,960	....	....	15,020	16,430	13,420	....	....	16,840	18,420	15,050	....	....
13	12,390	13,560	11,080	....	....	13,790	15,090	12,330	....	....	15,340	16,780	13,710	....	....
14	11,550	12,640	....	....	....	12,750	13,950	....	....	....	14,080	15,410	....	....	....
(b) CONCENTRATED LOADS															
2	....	....	....	....	56,420	....	....	....	....	....	....	....	....	....	....
3	....	....	....	43,020	31,050	....	....	....	....	46,170	....	....	....	59,060	42,630
4	47,470	51,940	42,440	29,890	21,580	....	....	59,760	42,090	30,380	....	....	....	41,940	30,280
5	36,860	40,330	32,950	23,210	16,750	49,630	54,300	44,360	31,250	22,560	....	....	59,550	32,210	23,250
6	30,410	33,270	27,190	19,150	13,820	39,500	43,220	35,310	24,870	17,950	51,150	55,960	45,720	26,020	18,790
7	26,040	28,490	23,280	16,400	11,840	32,850	35,940	29,370	20,680	14,930	41,330	45,220	36,950	21,790	15,730
8	22,850	25,010	20,430	14,390	10,390	28,160	30,810	25,170	17,730	12,800	34,600	37,860	30,930	18,720	13,510
9	20,410	22,330	18,240	12,850	9,280	24,660	26,980	22.040	15,530	11,210	29,720	32,520	26,570	16,390	11,830
10	18,470	20,210	16,510	11,630	8,390	21,940	24,010	19,620	13,820	9,970	26,030	28,480	23,270	14,570	....
11	16,880	18,470	15,090	10,630	....	19,790	21,650	17,690	12,460	....	23,140	25,320	20,690	....	....
12	15,560	17,020	13,910	....	....	18,020	19,710	16,110	....	....	20,840	22,800	18,630	....	....
13	14,430	15,790	12,900	....	....	16,540	18,100	14,790	....	....	18,940	20,720	16,930	....	....
14	13,470	14,740	....	....	....	15,300	16,740	....	....	....	17,360	18,990	....	....	....

beams were made. Finally, it was found that, if the critical stress, as ordinate, were plotted against the dimension ratio—

$$\frac{l\,d}{b\,t} \qquad (5)$$

—as abscissa, the resulting group of points would fall in such a pattern that a curve expressing the relationship could readily be plotted. Values of d, b, and t for rolled sections are tabulated in standard handbooks.

For each of the 132 beams, therefore, the ratio $\frac{l\,d}{b\,t}$ was computed. First the quantity $\frac{a\,d}{b\,t}$ was computed from the handbook values of these four factors, and then this value was multiplied by values of l/a from 2 to 14, to give a value of $\frac{l\,d}{b\,t}$ for each value of l/a.

A part of the resulting tabulation embracing only the five sections selected for Table 2, is given in Table 3. Only those $\frac{l\,d}{b\,t}$-values are shown for which

TABLE 3.—RATIO $\frac{l\,d}{b\,t}$, FOR THE BEAMS IN TABLE 2

l/a	14 WF 87	14 WF 78	14 WF 61	14 WF 43	14 WF 34
$\frac{a\,d}{b\,t}$	146.0	133.6	160.8	223.6	294.3
2	....	....	....	....	589
3	....	....	482	671	883
4	584	534	643	894	1,177
5	730	668	804	1,118	1,472
6	876	802	965	1,342	1,766
7	1,022	935	1,126	1,565	2,060
8	1,168	1,069	1,286	1,789	2,354
9	1,314	1,202	1,447	2,012	2,649
10	1,460	1,336	1,608	2,236	2,943
11	1,606	1,470	1,769	2,460	....
12	1,752	1,603	1,930	....	....
13	1,898	1,737	2,090	....	....
14	2,044	1,870	....	....	....

critical stresses are given in Table 2. The horizontal dividing lines in the two tables are the same.

The observations in Tables 2 and 3 for uniformly distributed load were then plotted in Fig. 2; values of $\frac{l\,d}{b\,t}$ were plotted as abscissas and the corresponding critical stresses f were plotted as ordinates. Round dots represent the five 14-in. sections, of the other points, × represents entries for all other beam sections for which l/b is less than 40; and +, those for which l/b is 40 or more. Not every entry can be shown, as in some instances they superimpose. It is obvious that the grouping of entries gives a good pattern and that a curve for a practical formula can easily be obtained from it. The formula for critical stress (Eq. 4) has the modulus of elasticity, E, as one of its variables, but there

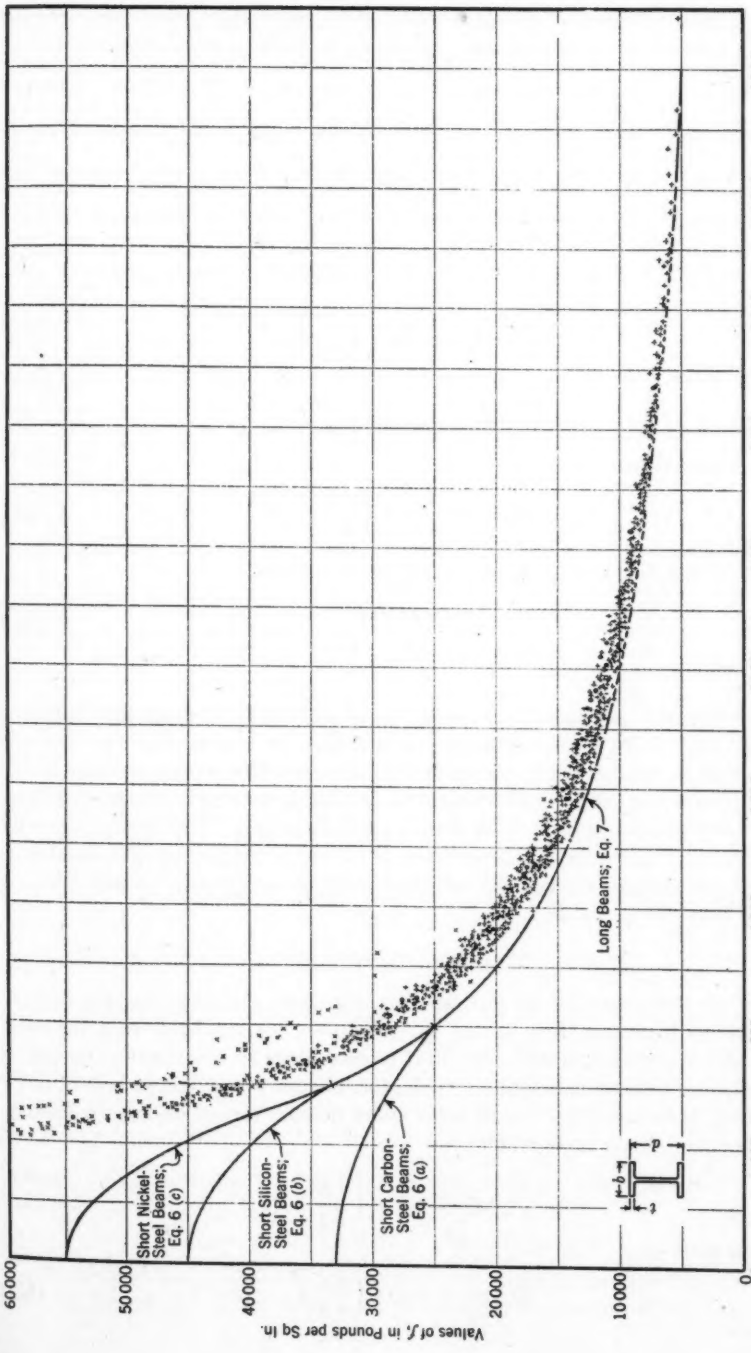

FIG. 2.—CRITICAL STRESSES COMPUTED FOR A SIMPLE BEAM, WITH A UNIFORMLY DISTRIBUTED LOAD AT THE TOP FLANGE, AND CURVES FOR THE FAILURE FORMULAS EQS. 6 AND 7

is no term representing the yield point strength of the steel. Therefore, as with the Euler curve for columns, the actual curve for critical stress must be cut at the yield point strength of the steel under consideration. For higher $\frac{l\,d}{b\,t}$-values (for long beams) then, the same formula should apply to all structural steels since E for all is close to 30,000,000 lb per sq in. For lower $\frac{l\,d}{b\,t}$-values (for short beams), the yield point of each of the three common structural steels is made the upper limit, and transition curves are inscribed.

Failure formulas for short beams (see Fig. 2) are: For carbon steel—

$$33{,}000 - 0.0125\left(\frac{l\,d}{b\,t}\right)^2 \quad \ldots\ldots (6a)$$

for silicon steel—

$$45{,}000 - 0.0324\left(\frac{l\,d}{b\,t}\right)^2 \quad \ldots\ldots (6b)$$

and, for nickel steel—

$$55{,}000 - 0.06\left(\frac{l\,d}{b\,t}\right)^2 \quad \ldots\ldots (6c)$$

For long beams, in carbon, silicon, and nickel steel alike:

$$\frac{20{,}000{,}000}{\frac{l\,d}{b\,t}} \quad \ldots\ldots (7)$$

These formulas are safe for beams which are laterally unsupported between their ends and where l is the span length; and also for beams with two or more panels—that is, beams which are prevented from rotation at one or more intermediate points and whose panel length is used as l, the unsupported length of the compression flange (as will be shown subsequently). The case of the uniformly loaded top flange has been made the basic example, since it results in lower critical stresses than the other five loading conditions, as can be seen from the k-values of Table 1.

Working Formulas

Working formulas can be written directly from the formulas for critical stress (failure formulas, Eqs. 6 and 7). Suppose, for instance, that for steel railway and highway structures the unit stresses usually specified for beams of zero length be retained and that safety factors comparable to those now in force are applied, then working formulas for short beams, corresponding to Eqs. 6, might be chosen: For carbon steel—

$$18{,}000 - 0.006\left(\frac{l\,d}{b\,t}\right)^2 \quad \ldots\ldots (8a)$$

for silicon steel—

$$24{,}000 - 0.014\left(\frac{l\,d}{b\,t}\right)^2 \quad \ldots\ldots (8b)$$

and, for nickel steel—

$$30{,}000 - 0.024\left(\frac{l\,d}{b\,t}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8c)$$

For long beams, of carbon, silicon, or nickel steel, the formula corresponding to Eq. 7 might be:

$$\frac{12{,}000{,}000}{\frac{l\,d}{b\,t}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Eqs. 8 and 9 have been plotted in Fig. 3 with transition points at $\frac{l\,d}{b\,t} = 1{,}000$, 700, and 500, respectively.

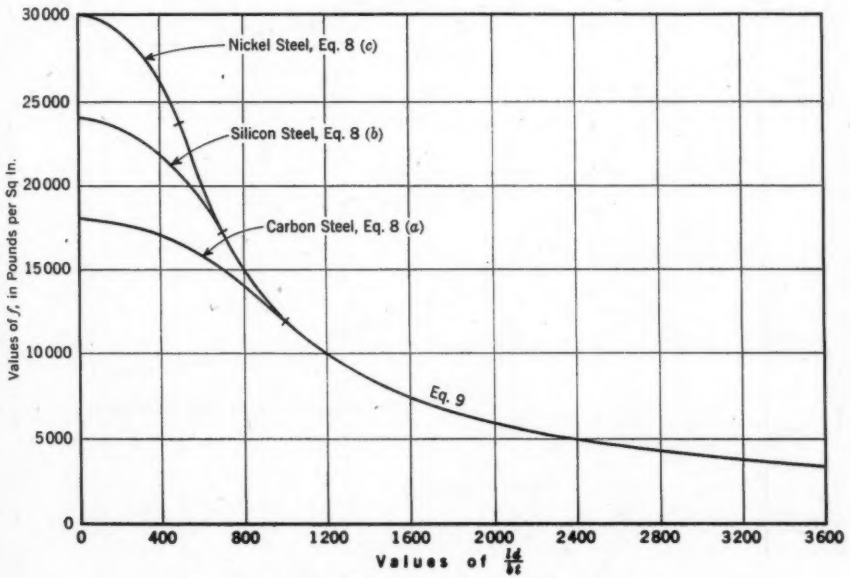

FIG. 3.—PROPOSED WORKING FORMULAS (EQS. 8 AND 9)

To simplify the use of these formulas, it is recommended that the $\frac{d}{b\,t}$-values be listed with the properties of beam sections in future editions of handbooks. Handbook tabulations of allowable stresses, giving "unit stresses" and "ratios" for converting allowable loads on laterally supported beams to allowable loads on laterally unsupported beams, based on the l/b ratio, could be replaced by tabulations of allowable stresses based on the $\frac{l\,d}{b\,t}$-ratio.

THE FIVE OTHER TYPES OF LOADING

Working formulas of the type of Eqs. 8 and 9, which are based on the failure formulas Eqs. 6 and 7, are proposed for incorporation in specifications. They

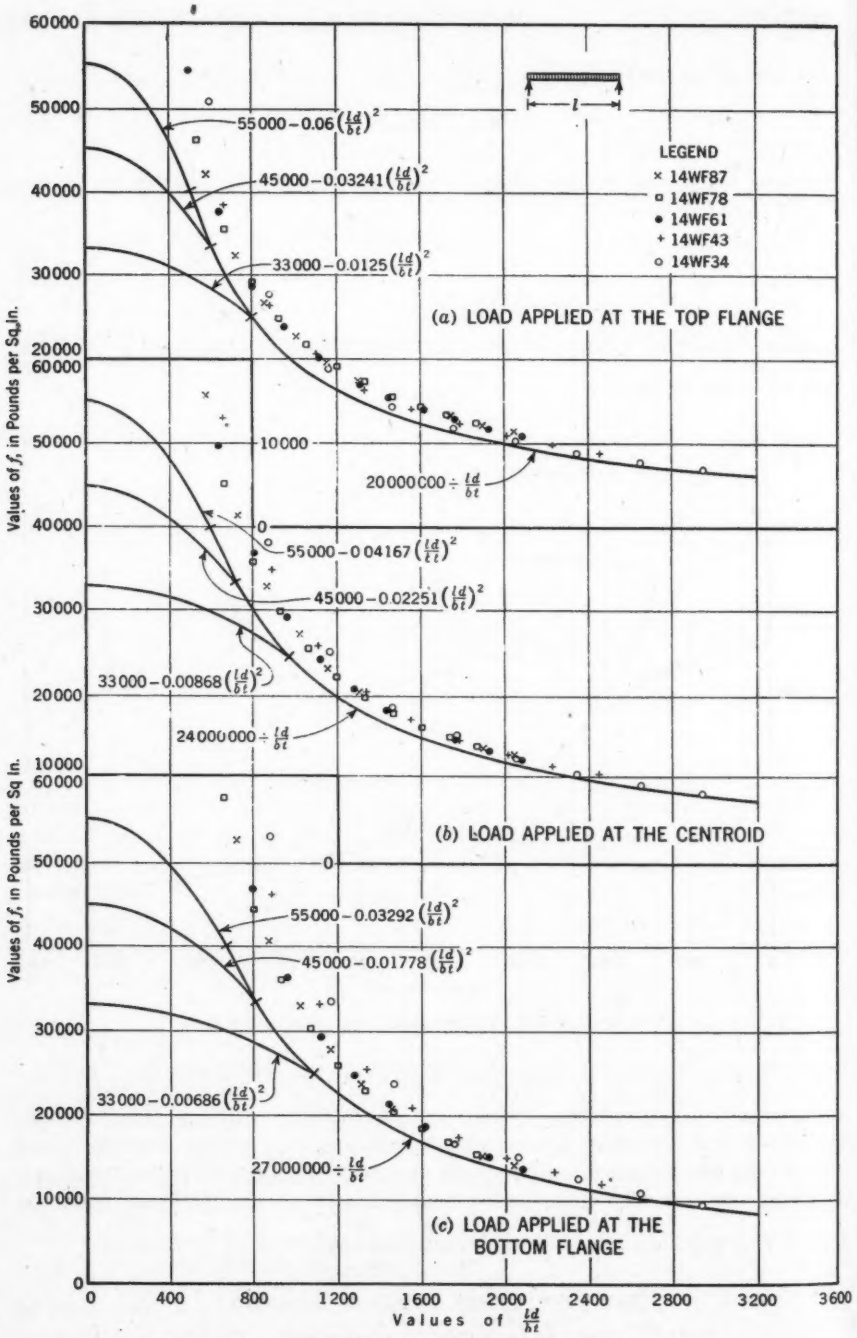

FIG. 4.—CRITICAL STRESSES FOR A SIMPLE BEAM, UNIFORMLY LOADED
(*a*) Load Applied at the Top Flange
(*b*) Load Applied at the Centroid
(*c*) Load Applied at the Bottom Flange

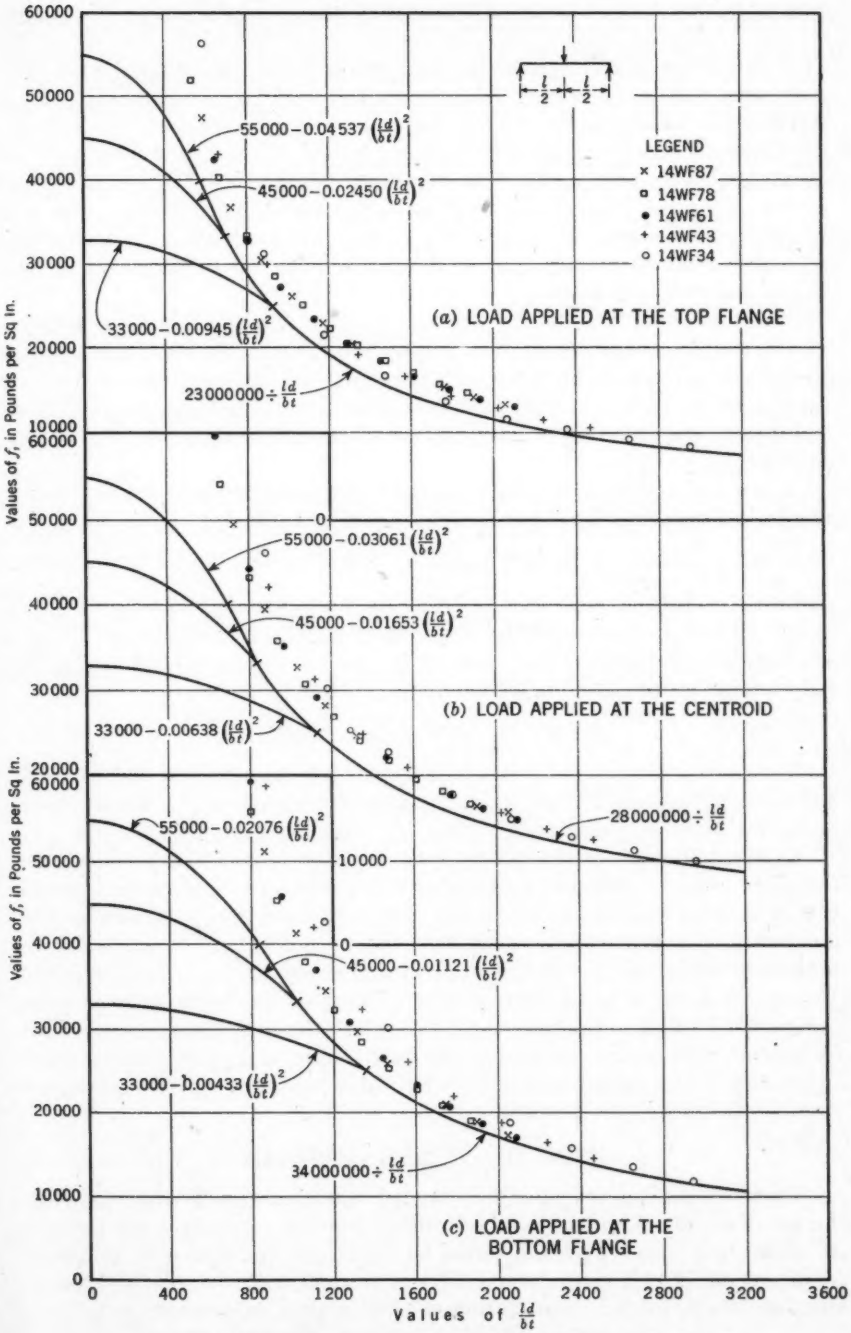

FIG. 5.—CRITICAL STRESSES FOR A SIMPLE BEAM, CONCENTRATED LOAD
(*a*) Load Applied at the Top Flange
(*b*) Load Applied at the Centroid
(*c*) Load Applied at the Bottom Flange

are safe for the most severe of the six types of loading considered—the other five types will be of interest for investigators and for special problems. In erection, for example, where long beams frequently have to be picked, and placed temporarily without intermediate lateral support, the weight of the beam itself is the only loading—that is, the condition is that of a uniform load at the centroid. This condition will allow the use of higher failure stress and, therefore, also higher working stress. Other special loadings, which might represent a combination of any of the six loadings, or an intermediate case, would require interpolation.

The five 14-in. wide-flange sections, which had been chosen for Tables 2 and 3, and for which the critical stresses are included in Fig. 2, seem to be characteristic of the entire group of 132 beams; and these sections only will be used in the derivation of failure curves for the remaining five types of loading. The five beams selected all have depths of 14 in. and flange widths varying from $14\frac{1}{2}$ in. to $6\frac{3}{4}$ in.

Using Eq. 4 with the k-values for the five loading conditions from Table 1, the critical stresses are computed and entered in Table 2.

If the $\frac{l}{b}\frac{d}{t}$-values of Table 3 again are taken as abscissas and the critical stresses f in Table 2 are taken as ordinates, curves similar to those in Fig. 2 result. These are shown in Figs. 4 and 5. Fig. 4(a) is a repetition of Fig. 2, showing the calculated critical stresses for the five typical beams used. The failure formulas are listed in Table 4.

PART II. TWO-BAY BEAMS

For a beam with lateral restraint at the middle cross section and a concentrated load P applied at that point, and also for the case of a uniformly loaded beam, the k-values, presented in Table 1(c) have been computed.

In the derivation of the failure formulas, Eqs. 6 and 7, and the working formulas, Eqs. 8 and 9, the k-values of Table 1(b), Col. 5, for a beam loaded with a uniform load at the top flange were used since they were smaller than the k-values for any other loading condition considered. Comparison of the tabulated k-values of Table 1, Col. 5, with those of Table 1(c) indicates that any error induced by using the unsupported length of the beam in the proposed failure and working formulas is on the safe side. Computations were also made for a beam with lateral restraint at the third points, and at the quarter points and center. The results consistently indicated the safety of the assumption that l is the unsupported length.

PART III. PLATE GIRDERS

The foregoing close investigation of rolled beams has been facilitated through the use of handbook values for the torsional bending constant a, the formulas for which had been set up and tested by Inge Lyse and Bruce G. Johnston,[9] Members, ASCE. Torsion formulas for plate girders do not seem to be available, and the following investigation of plate girders, of necessity, will be an

[9] "Structural Beams in Torsion," by Inge Lyse and Bruce G. Johnston, *Transactions*, ASCE, Vol. 101, 1936, p. 857.

TABLE 4.—FAILURE FORMULAS FOR SIMPLE BEAMS

Load at:	Steel	Short beams (see Figs. 4 and 5)	Transition $\frac{l\,d}{b\,t}$	Transition f	Long beams (see Figs. 4 and 5)
(1)	(2)	(3)	(4)	(5)	(6)
(a) Uniformly Distributed Load					
Top flange	Carbon	$33{,}000-0.01250\left(\frac{l\,d}{b\,t}\right)^2$	800	25,000	$\frac{20{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000-0.03241\left(\frac{l\,d}{b\,t}\right)^2$	600	33,333	
	Nickel	$55{,}000-0.06000\left(\frac{l\,d}{b\,t}\right)^2$	500	40,000	
Centroid	Carbon	$33{,}000-0.00868\left(\frac{l\,d}{b\,t}\right)^2$	960	25,000	$\frac{24{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000-0.02251\left(\frac{l\,d}{b\,t}\right)^2$	720	33,333	
	Nickel	$55{,}000-0.04167\left(\frac{l\,d}{b\,t}\right)^2$	600	40,000	
Bottom flange	Carbon	$33{,}000-0.00686\left(\frac{l\,d}{b\,t}\right)^2$	1,080	25,000	$\frac{27{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000-0.01778\left(\frac{l\,d}{b\,t}\right)^2$	810	33,333	
	Nickel	$55{,}000-0.03292\left(\frac{l\,d}{b\,t}\right)^2$	675	40,000	
(b) Concentrated Load at the Center of the Span					
Top flange	Carbon	$33{,}000-0.00945\left(\frac{l\,d}{b\,t}\right)^2$	920	25,000	$\frac{23{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000-0.02450\left(\frac{l\,d}{b\,t}\right)^2$	690	33,333	
	Nickel	$55{,}000-0.04537\left(\frac{l\,d}{b\,t}\right)^2$	575	40,000	
Centroid	Carbon	$33{,}000-0.00638\left(\frac{l\,d}{b\,t}\right)^2$	1,120	25,000	$\frac{28{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000-0.01653\left(\frac{l\,d}{b\,t}\right)^2$	840	33,333	
	Nickel	$55{,}000-0.03061\left(\frac{l\,d}{b\,t}\right)^2$	700	40,000	
Bottom flange	Carbon	$33{,}000-0.00433\left(\frac{l\,d}{b\,t}\right)^2$	1,360	25,000	$\frac{34{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000-0.01121\left(\frac{l\,d}{b\,t}\right)^2$	1,020	33,333	
	Nickel	$55{,}000-0.02076\left(\frac{l\,d}{b\,t}\right)^2$	850	40,000	

approximation only. Consider a plate girder with a section such as that in Fig. 6(*a*), for which the moments of inertia are given in Table 5. Assuming that the torsional formulas for rolled beams apply also to plate girders, and that the flange rivets join the flange segments effectively, the torsion constant K

can be computed thus:[10]

$$2 \times 0.333 \times 13 \times 3^3 = 234.0$$
$$2 \times 0.333 \times 5.25 \times 2.5^3 = 54.7$$
$$8 \times 0.333 \times 3.5 \times 1^3 = 9.3$$
$$8 \times 0.333 \times 1.75 \times 1^3 = 4.7$$
$$1 \times 0.333 \times 88 \times 0.5^3 = 3.7$$
$$2 \times 0.109 \times 2.15 = 46.9$$
$$16 \times 0.10504 \times 1^4 = -1.7$$

Torsion constant K (in.^{4}) = 351.6

TABLE 5.—MOMENT OF INERTIA, PLATE GIRDER IN FIG. 6(*a*)

Section shapes	Number of pieces	Dimensions (inches)	MOMENT OF INERTIA (In.^{4})	
			I_x	I_y
Outer cover plates	Two	20 by 1	107,126	1,333
Inner cover plates	Two	20 by 1	103,026	1,333
Angles	Four	8 by 8 by 1	137,906	768
Web	One	100 by $\frac{1}{2}$	41,667	1
Total	....	....	389,725	3,435

In a report published in 1941, Ingvald E. Madsen, Jun. ASCE, assumes[11] that the torsional constant of a riveted girder should be one third of the K-value for an identical solid section, because of rivet slip. If this assumption were used in the foregoing computation, it would reduce the K-value of a solid section (which has been computed separately and found to be 428 in.^{4}) to 142.7 in.^{4}, and is thus considerably less than the foregoing value of 351.6 in.^{4} The writer believes that the effect of rivet slip on the small specimens tested by Mr. Madsen is much greater than on full-size plate girders, and he has therefore rejected Mr. Madsen's assumption in favor of the foregoing computations.

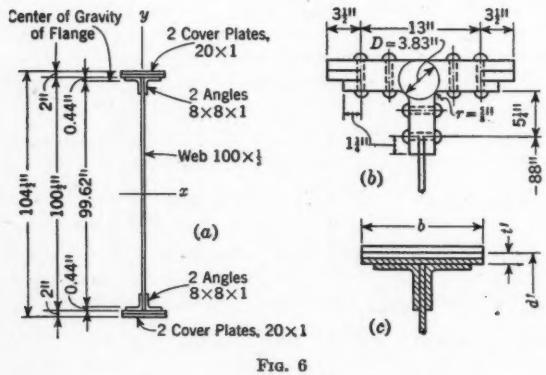

FIG. 6

To determine the torsional bending constant a, the formula—[12]

$$a = 0.806\, d_c \sqrt{\frac{I_y}{K}} \qquad (10)$$

[10] "Structural Beams in Torsion," by Inge Lyse and Bruce G. Johnston, *Transactions*, ASCE, Vol. 101, 1936, p. 865, Eq. 21.

[11] "Report of Crane Girder Tests," by I. Madsen, *Iron and Steel Engineer*, November, 1941, Section 4, p. 68.

[12] "Structural Beams in Torsion," by Inge Lyse and Bruce G. Johnston, *Transactions*, ASCE, Vol. 101, 1936, p. 869.

—gives $a = 0.806 \times 99.62 \times \sqrt{\frac{3{,}435}{351.6}} = 251.0$ in. and the value of $10{,}000 \frac{I_y d^2}{I_x a^2}$ is $10{,}000 \times \frac{3{,}435 \times 104.5^2}{389{,}725 \times 251.0^2} = 15.28$. In Eq. 10, d_c is the depth of the plate girder between flange centroids.

Since the proposed specification formulas Eqs. 8 and 9 involve the thickness of the flange, t, an equivalent thickness of the flange must be evaluated in the case of plate girders. For rolled beams, t is tabulated in handbooks, but can also be expressed by the formula:

$$t = \frac{6 I_y}{b^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

If b is assumed to be the maximum flange width of the plate girder, Eq. 11 could be used to determine the equivalent average flange thickness of the plate girder of Fig. 6, for which, then, $t = \frac{6 \times 3{,}435}{20^3} = 2.576$ in., and the ratio $\frac{a\,d}{b\,t} = \frac{251.0 \times 104.5}{20 \times 2.576} = 509.1$.

Table 6(a) has been compiled from these values. In Table 6, Col. 2 gives the failure stresses of the plate girder up to $\frac{l}{b} = \frac{l}{a} \times \frac{a}{b} = \frac{8 \times 251.0}{20} = 100.4$, for a uniformly distributed load applied at the top flange computed as for Table 2(a); Col. 3, the $\frac{l\,d}{b\,t}$-ratio derived from the $\frac{a\,d}{b\,t}$-values; Col. 4, the values of the failure curve of Eq. 7; and Col. 5, the ratio of failure stress to failure curve. Table 6(a) shows that the failure stresses of Eq. 7 will be exceeded for $\frac{l\,d}{b\,t}$-ratios greater than about 2,000, or a laterally unsupported girder length of $l = \frac{l\,d}{b\,t} \times \frac{b\,t}{d} = 2{,}000 \frac{20 \times 2.576}{104.5} = 986$ in., or 82.2 ft.

This small overstress for long lengths seems to be immaterial; but, since close investigations and tests are not available, it is proposed to use, instead of Eq. 11, the following formula for the computations of average flange thicknesses of plate girders:

$$t = \frac{5 I_y}{b^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which I_y is the moment of inertia of the girder about the y-axis; and b is the maximum flange width. Table 6(b) has been prepared to show the results of using Eq. 12 instead of Eq. 11. If the compression flange of a plate girder is different from the tension flange, twice the moment of inertia of the compression flange about the y-axis should be used as I_y in Eq. 12.

Long plate girders usually have cover plates of various lengths, and an average moment of inertia I_y of the girder will have to be computed to express its lateral stiffness. If the cover plates are computed to follow a parabolic moment diagram, with a minimum total cover plate thickness of t (min) at the ends, and a maximum total cover plate thickness t (max) in the middle, then

the effective average thickness t' (Fig. 6(*c*)) will be

$$t' = t(\min) + \tfrac{2}{3}\,[t(\max) - t(\min)] \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

This effective average flange thickness t' will have to be used for determining I_y in Eq. 12; and, with the effective depth d' (Fig. 6(*c*)), the $\frac{l\,d}{b\,t}$-ratio can be established.

TABLE 6.—COMPARISON OF FAILURE STRESS WITH THE FAILURE CURVE FOR THE PLATE GIRDER IN FIG. 6(*a*), USING DIFFERENT EQUIVALENT FLANGE THICKNESSES, t

$\frac{l}{a}$	Failure stress, f	(*a*) $t = 6\,I_y/b^3$ (Eq. 11)			(*b*) $t = 5\,I_y/b^3$ (Eq. 12)		
		$\frac{l\,d}{b\,t}$	$\frac{20{,}000{,}000}{\frac{l\,d}{b\,t}}$	$\frac{\text{Col. 2}}{\text{Col. 4}}$	$\frac{l\,d}{b\,t}$	$\frac{20{,}000{,}000}{\frac{l\,d}{b\,t}}$	$\frac{\text{Col. 2}}{\text{Col. 7}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
2	26,000	1,018	19,650	1.323	1,222	16,370	1.588
3	14,200	1,527	13,100	1.084	1,832	10,920	1.300
4	9,800	2,036	9,820	0.998	2,443	8,190	1.197
5	7,560	2,546	7,860	0.962	3,054	6,550	1.154
6	6,200	3,055	6,550	0.947	3,665	5,460	1.136
7	5,290	3,564	5,610	0.943	4,276	4,680	1.130
8	4,630	4,073	4,910	0.943	4,886	4,090	1.132

The computation of the effective average flange thickness would not be necessary for conditions which are generally encountered. Such a computation applies and is of consequence only for long unsupported lengths—as, for example, for the investigation of erection stresses of plate girders temporarily unsupported laterally. It should be noted that the average flange thickness of Eq. 12 should be used for all compound sections, such as cover-plated rolled beams, since a condition results which is similar to that in the plate girder which has been investigated.

PART IV. CONCLUSION

By investigating the strength of steel beams in lateral buckling, it has been shown that the critical stresses of rolled beams can be grouped to follow a good pattern (Fig. 2) and that working formulas can be derived from them. Working formulas (Eqs. 8 and 9) are proposed for inclusion in specifications. For plate girders and other compound sections it is proposed that the same formulas be used with l equal to the unsupported length of the compression flange, d equal to the depth of the section, b equal to the maximum compression flange width, and an average compression flange thickness (see Eq. 12) of $t = 5\,I_y/b^3$. Additional failure formulas for special loading conditions are derived and the treatment of long plate girders is discussed.

For all computations it is assumed that the beams at their supports are held in vertical positions. Beams with eccentrically applied loads are torsional problems and have not been included in this paper.

Commonly used beam sections and plate girders have been discussed. Channels, zees, angles, and other unsymmetrical sections are seldom used as

long beams, and it is proposed to allow the same unit stress for them as for symmetrical sections. The deviation of stresses of the proposed formulas from those of other beam formulas is negligible for short beams, and these unsymmetrical sections do not deserve special consideration.

ACKNOWLEDGMENT

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APPENDIX I—DERIVATION OF *k*-VALUES

SIMPLE BEAMS; CONCENTRATED LOAD AT CENTER

The solution of Eq. 3 involves the evaluation of quantities of the angle of twist β. This can most readily be done by expressing β as a series:[13]

$$\beta = a_1 \cos\frac{\pi z}{l} + a_3 \cos\frac{3\pi z}{l} + \cdots \quad (14)$$

Only the first two terms of the series are of sufficient magnitude to be of importance. Introducing $y = \frac{a_3}{a_1}$, the following values are obtained:

$$\beta^2{}_o = a^2{}_1 (1 + y)^2 \quad (15a)$$

$$\int_0^{l/2} \beta^2 \left(\frac{l}{2} - z\right)^2 dz = a^2{}_1 l^3 \frac{1}{8}\left(\frac{1}{6} y^2 + \frac{1}{9\pi^2} y^2 + \frac{5}{2\pi^2} y + \frac{1}{6} + \frac{1}{\pi^2}\right) \quad (15b)$$

$$\int_0^{l/2} \left(\frac{d\beta}{dz}\right)^2 dz = a^2{}_1 \frac{1}{l}\frac{\pi^2}{4}(1 + 9y^2) \quad (15c)$$

and

$$\int_0^{l/2} \left(\frac{d^2\beta}{dz^2}\right)^2 dz = a^2{}_1 \frac{1}{l^3}\frac{\pi^4}{4}(1 + 81y^2) \quad (15d)$$

Substituting Eqs. 15 in Eq. 3:

$$P = \frac{4 E I_v d}{l^3}\left\{ \mp (1 + y)^2 + \sqrt{(1 + y)^4 + \frac{\pi^2}{8}\left[\frac{y^2}{6} + \frac{y^2}{9\pi^2} + \frac{5y}{2\pi^2} + \frac{1}{6} + \frac{1}{\pi^2}\right] \times \left[\frac{l^2}{a^2}(1 + 9y^2) + \pi^2 (1 + 81y^2)\right]}\right\} \div \left[\frac{y^2}{6} + \frac{y^2}{9\pi^2} + \frac{5y}{2\pi^2} + \frac{1}{6} + \frac{1}{\pi^2}\right] \quad (16)$$

[13] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 266, Eq. (*f*).

Eq. 16 will now be transformed into an equation in unit stress f, such that:

$$f = \frac{P\,l}{4} \times \frac{d}{2\,I_x} \quad \ldots\ldots (17)$$

for any beam, in which I_x is the moment of inertia about the x-axis. If Eq. 17 is written:

$$f = \frac{I_y\,d^2}{I_x\,a^2}\,E\,\frac{a^2}{l^2}\,\frac{1}{2}\,\frac{P\,l^3}{4\,E\,I_y\,d} \quad \ldots\ldots (18)$$

and, if also the substitution—

$$A = \frac{1}{6}\,y^2 + \frac{1}{9\,\pi^2}\,y^2 + \frac{5}{2\,\pi^2}\,y + \frac{1}{6} + \frac{1}{\pi^2} \quad \ldots\ldots (19)$$

—is made, the unit stress becomes

$$f = \frac{I_y\,d^2}{I_x\,a^2}\,E\,\frac{a^2}{l^2}\,\frac{1}{2\,A}$$

$$\times \left\{ \mp (1+y)^2 + \sqrt{(1+y)^4 + \frac{\pi^2}{8}\,A\left[\frac{l^2}{a^2}(1+9\,y^2) + \pi^2(1+81\,y^2)\right]} \right\} \quad \ldots (20)$$

or in its general form, Eq. 4, in which k varies because of the introduction of $\frac{P\,d}{4}\beta^2{}_o$ in Eq. 2. For a concentrated load applied, respectively: At the top flange—

$$k = \frac{1}{2\,A}\left\{ -(1+y)^2 + \sqrt{(1+y)^4 + \frac{\pi^2}{8}\,A\left[\frac{l^2}{a^2}(1+9\,y^2) + \pi^2(1+81\,y^2)\right]} \right\} \quad \ldots (21a)$$

at the centroid—

$$k = \frac{\pi}{4\,A}\sqrt{\frac{A}{2}\left[\frac{l^2}{a^2}(1+9\,y^2) + \pi^2(1+81\,y^2)\right]} \quad \ldots\ldots (21b)$$

and, at the bottom flange—

$$k = \frac{1}{2\,A}\left\{ (1+y)^2 + \sqrt{(1+y)^4 + \frac{\pi^2}{8}\,A\left[\frac{l^2}{a^2}(1+9\,y^2) + \pi^2(1+81\,y^2)\right]} \right\} \quad \ldots (21c)$$

The unit stress determined from Eq. 4 is a variable depending on the value of k. Since the basic problem is to determine the least value of f which will cause the beam to buckle (that is, "the critical stress"), it is necessary to determine, for each of a series of possible ratios l/a the value of y that will make the value of k a minimum. These critical values of y have been computed, and are given in Table 7(a). The corresponding computed values of k for these critical values of y have been tabulated in Table 1(a).

Simple Beams; Uniformly Distributed Load

Assuming a uniformly distributed load of w per linear unit, as in Fig. 7(a), equations similar to those for concentrated load can be set up. For work done by the external forces, assumed to be applied at the centroid, a method anal-

TABLE 7.—Critical Values of y for Use in Computations of k (See Table 1)

$\frac{l}{a}$	Simple Beams						(c) Two-Bay Beams			
	(a) Concentrated Load, Eq. 21			(b) Uniform Load, Eq. 36			Concentrated Load, Eq. 43	Uniform Load, Eq. 50		
	Top flange	Centroid	Bottom flange	Top flange	Centroid	Bottom flange		Top flange	Centroid	Bottom flange
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2	0.014	0.008	−0.005	0.002	0.005	0.010	−0.052	−0.030	−0.036	−0.046
3	0.016	0.010	−0.004	0.004	0.006	0.012	−0.066	−0.038	−0.046	−0.054
4	0.020	0.014	−0.001	0.005	0.008	0.014	−0.080	−0.046	−0.054	−0.064
5	0.024	0.016	0.002	0.006	0.010	0.016	−0.092	−0.056	−0.064	−0.072
6	0.028	0.020	0.005	0.008	0.012	0.018	−0.104	−0.064	−0.072	−0.080
7	0.032	0.022	0.008	0.010	0.014	0.021	−0.116	−0.072	−0.080	−0.088
8	0.036	0.026	0.012	0.012	0.016	0.023	−0.126	−0.078	−0.086	−0.094
9	0.038	0.028	0.015	0.014	0.018	0.024	−0.132	−0.084	−0.090	−0.098
10	0.042	0.031	0.018	0.015	0.020	0.026	−0.140	−0.088	−0.096	−0.102
11	0.044	0.034	0.020	0.016	0.022	0.028	−0.146	−0.092	−0.098	−0.106
12	0.046	0.036	0.022	0.018	0.023	0.029	−0.148	−0.096	−0.102	−0.108
13	0.048	0.038	0.026	0.020	0.024	0.030	−0.152	−0.098	−0.104	−0.110
14	0.048	0.040	0.028	0.021	0.025	0.031	−0.156	−0.102	−0.106	−0.112

ogous to that used by Professor Timoshenko[14] can be used. Consider an element of the longitudinal axis at point D and consider the cross section mn at that point fixed. Then, because of bending of this element in the (ξ, ζ)-plane,

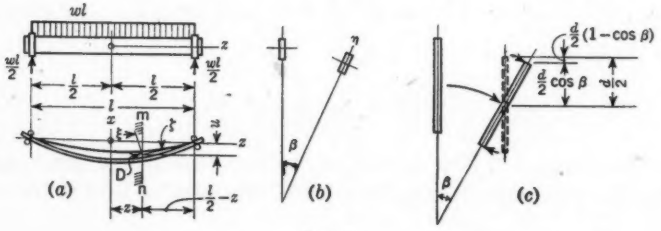

Fig. 7

the end of the beam, where the reaction ($= w\,l/2$) is applied, describes an infinitely small arc,

$$ds = \frac{d^2u}{dz^2}\left(\frac{l}{2} - z\right)dz \qquad (22a)$$

in the (ξ, ζ)-plane, the vertical component of which is

$$ds_v = \beta\,\frac{d^2u}{dz^2}\left(\frac{l}{2} - z\right)dz \qquad (22b)$$

[14] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 253.

In Eqs. 22, u is the deflection of any element. The uniform load between point D and the right-hand end of the beam can be considered concentrated at its center of gravity, at a distance $\frac{1}{2}(l/2 - z)$ from point D, and the vertical component of movement, due to the bending of the element at point D, is

$$ds_D = \beta \frac{d^2u}{dz^2} \frac{1}{2} \left(\frac{l}{2} - z \right) dz \ldots\ldots\ldots\ldots\ldots\ldots (22c)$$

The work done by the external forces to the right of point D, due to bending of the element at point D caused by beam buckling, is

$$V_D = \frac{w\,l}{2} \beta \frac{d^2u}{dz^2} \left(\frac{l}{2} - z \right) dz - w \left(\frac{l}{2} - z \right) \beta \frac{d^2u}{dz^2} \frac{1}{2} \left(\frac{l}{2} - z \right) dz$$

$$= \frac{w}{2} \beta \frac{d^2u}{dz^2} \left(\frac{l^2}{4} - z^2 \right) dz \ldots\ldots (23)$$

and the work done on the full length of the beam is twice the summation of these forces from 0 to $l/2$:

$$V_1 = w \int_0^{l/2} \beta \frac{d^2u}{dz^2} \left(\frac{l^2}{4} - z^2 \right) dz \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

If the load is applied at the top or bottom flanges, instead of along the centroid of the beam, the external work done by the load is increased or decreased by the movement of the flanges caused by the beam rotation.[15] Considering the beam of Fig. 7(*a*) as a simple beam between its reactions, an element at the point of load application at the top or the bottom of the beam (Fig. 7(*c*)) is either lowered or raised a distance $\frac{d}{2}(1 - \cos \beta)$ or approximately $\frac{d}{4}\beta^2$. The work done by the element then is $\pm \frac{w\,d}{4} \beta^2\, dz$, and the work done along the full length of the beam,

$$V_2 = \pm \frac{w\,d}{2} \int_0^{l/2} \beta^2\, dz \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The strain energy of lateral bending, torsion, and flange twist can be expressed in the same form as for a beam with concentrated load in the middle,[16] thus:

$$w \int_0^{l/2} \beta \frac{d^2u}{dz^2} \left(\frac{l^2}{4} - z^2 \right) dz \pm \frac{w\,d}{2} \int_0^{l/2} \beta^2\, dz$$

$$= B \int_0^{l/2} \left(\frac{d^2u}{dz^2} \right)^2 dz + C \int_0^{l/2} \left(\frac{d\beta}{dz} \right)^2 dz + \frac{B\,d^2}{4} \int_0^{l/2} \left(\frac{d^2\beta}{dz^2} \right)^2 dz \ldots (26)$$

Equations corresponding to those for a concentrated load, given by Professor Timoshenko[17] can be written for a uniformly distributed load, for which the

[15] "The Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 254.

[16] *Ibid.*, p. 266, Eq. (*d*).

[17] *Ibid.*, p. 251.

vertical moment is

$$M_x = -\frac{w\,l}{2}\left(\frac{l}{2} - z\right) + \frac{w}{2}\left(\frac{l}{2} - z\right)^2 = -\frac{w}{2}\left(\frac{l^2}{4} - z^2\right) \ldots\ldots(27a)$$

This moment, projected to the η-axis, is

$$M_\eta = \frac{w}{2}\left(\frac{l^2}{4} - z^2\right)\beta \ldots\ldots(27b)$$

and the differential equation for the buckled beam is

$$B\,\frac{d^2u}{dz^2} = \frac{w}{2}\left(\frac{l^2}{4} - z^2\right)\beta \ldots\ldots(27c)$$

Using the value of $\frac{d^2u}{dz^2}$ of Eq. 27*c*, the equation for uniformly distributed load, corresponding to Eq. 2, will be:

$$\frac{w^2}{4\,E\,I_y}\int_0^{l/2}\beta^2\left(\frac{l^2}{4} - z^2\right)^2 dz \pm \frac{w\,d}{2}\int_0^{l/2}\beta^2\,dz - \frac{E\,I_y}{4}\frac{l^2}{a^2}\frac{d^2}{l^2}\int_0^{l/2}\left(\frac{d\beta}{dz}\right)^2 dz - \frac{E\,I_y}{4}d^2\int_0^{l/2}\left(\frac{d^2\beta}{dz^2}\right)^2 dz = 0 \ldots(28)$$

This equation may be solved for the loading $w\,l$:

$$w\,l = E\,I_y\,dl\left\{\mp\int_0^{l/2}\beta^2\,dz + \sqrt{\int_0^{l/2}(\beta^2\,dz)^2 + \int_0^{l/2}\beta^2\left(\frac{l^2}{4} - z^2\right)^2 dz\left[\frac{l^2}{a^2}\frac{1}{l^2}\int_0^{l/2}\left(\frac{d\beta}{dz}\right)^2 dz + \int_0^{l/2}\left(\frac{d^2\beta}{dz^2}\right)^2 dz^2\right]}\right\} \div \left[\int_0^{l/2}\beta^2\left(\frac{l^2}{4} - z^2\right)^2 dz\right] \ldots(29)$$

With the same definitions for β and y as for concentrated load and the additional solutions for the integrals—

$$\int_0^{l/2}\beta^2\,dz = a^2{}_1\,\frac{l}{4}\,(1 + y^2) \ldots\ldots(30a)$$

and

$$\int_0^{l/2}\beta^2\left(\frac{l^2}{4} - z^2\right)^2 dz = a^2{}_1\,l^5\,\frac{1}{8}\left(\frac{1}{15}\,y^2 + \frac{1}{27\,\pi^4}\,y^2 + \frac{45}{8\,\pi^4}\,y + \frac{1}{15} + \frac{3}{\pi^4}\right) \ldots(30b)$$

—the loading may be expressed thus:

$$w\,l = \frac{2\,E\,I_y\,d}{l^3}\left\{\mp(1+y^2) + \sqrt{(1+y^2)^2 + \frac{\pi^2}{2}\left[\frac{y^2}{15} + \frac{y^2}{27\,\pi^4} + \frac{45\,y}{8\,\pi^4} + \frac{1}{15} + \frac{3}{\pi^4}\right]} \times \sqrt{\left[\frac{l^2}{a^2}(1+9\,y^2) + \pi^2(1+81\,y^2)\right]}\right\} \div \left[\frac{y^2}{15} + \frac{y^2}{27\,\pi^4} + \frac{45\,y}{8\,\pi^4} + \frac{1}{15} + \frac{3}{\pi^4}\right] \quad \ldots (31)$$

Eq. 31 may be transformed into an equation in unit stress f, by a process similar to that used in the case of concentrated load (Eq. 17):

$$f = \frac{w\,l^2}{8} \times \frac{d}{2\,I_x} \quad \ldots (32)$$

or

$$f = \frac{I_y\,d^2}{I_x\,a^2}\,E\,\frac{a^2}{l^2}\,\frac{1}{8}\,\frac{w\,l\,l^3}{2\,E\,I_y\,d} \quad \ldots (33)$$

and, with the substitution—

$$A = \frac{1}{15}\,y^2 + \frac{1}{27\,\pi^4}\,y^2 + \frac{45}{8\,\pi^4}\,y + \frac{1}{15} + \frac{3}{\pi^4} \quad \ldots (34)$$

—the unit stress becomes (compare Eq. 20):

$$f = \frac{I_y\,d^2}{I_x\,a^2}\,E\,\frac{a^2}{l^2}\,\frac{1}{8\,A}\left\{\mp(1+y^2) + \sqrt{(1+y^2)^2 + \frac{\pi^2}{2}\,A\left[\frac{l^2}{a^2}(1+9\,y^2) + \pi^2(1+81\,y^2)\right]}\right\} \quad \ldots (35)$$

Eq. 35 has the general form of Eq. 4. Values of k are, for a uniformly distributed load applied, respectively: At the top flange,

$$k = \frac{1}{8\,A}\left\{-(1+y^2) + \sqrt{(1+y^2)^2 + \frac{\pi^2}{2}\,A\left[\frac{l^2}{a^2}(1+9\,y^2) + \pi^2(1+81\,y^2)\right]}\right\} \quad \ldots (36a)$$

at the centroid,

$$k = \frac{\pi}{8\,A}\sqrt{\frac{A}{2}\left[\frac{l^2}{a^2}(1+9\,y^2) + \pi^2(1+81\,y^2)\right]} \quad \ldots (36b)$$

and, at the bottom flange,

$$k = \frac{1}{8A}\left\{(1+y^2) + \sqrt{(1+y^2)^2 + \frac{\pi^2}{2}A\left[\frac{l^2}{a^2}(1+9y^2) + \pi^2(1+81y^2)\right]}\right\} \ldots\ldots (36c)$$

To determine the least value of f in Eq. 4, the critical values of y for a given ratio of l/a have been computed, and are listed in Table 7(*b*). The corresponding computed values of k for these critical values of y are given in Table 1(*b*).

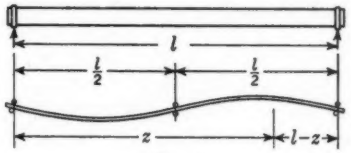

FIG. 8

TWO-BAY BEAMS; CONCENTRATED LOAD AT CENTER

For a beam with lateral restraint at the middle cross section and with a load P applied at that point (Fig. 8), an equation similar to Eq. 2 can be written:

$$\frac{P^2}{4EI_y}\int_{l/2}^{l}\beta^2(l-z)^2\,dz - \frac{EI_y}{8}\frac{l^2}{a^2}\frac{d^2}{l^2}\int_0^l\left(\frac{d\beta}{dz}\right)^2 dz - \frac{EI_y}{8}d^2\int_0^l\left(\frac{d^2\beta}{dz^2}\right)^2 dz = 0 \ldots\ldots (37)$$

Note that the term in Eq. 2, which takes into account the external work done by the load P due to twisting of the middle cross section, disappears for Eq. 37. Solved for P, Eq. 37 gives

$$P = EI_y d\sqrt{\frac{\frac{l^2}{a^2}\frac{1}{l^2}\int_0^l\left(\frac{d\beta}{dz}\right)^2 dz + \int_0^l\left(\frac{d^2\beta}{dz^2}\right)^2 dz}{2\int_{l/2}^{l}\beta^2(l-z)^2\,dz}} \ldots\ldots (38)$$

in which

$$\beta = a_2\sin\frac{2\pi z}{l} + a_4\sin\frac{4\pi z}{l} \ldots\ldots (39a)$$

and

$$y = \frac{a_4}{a_2} \ldots\ldots (39b)$$

The integrals in Eq. 38 are solved thus:

$$\int_0^l\left(\frac{d\beta}{dz}\right)^2 dz = a_2^2\frac{1}{l}2\pi^2(1+4y^2) \ldots\ldots (40a)$$

$$\int_0^l\left(\frac{d^2\beta}{dz^2}\right)^2 dz = a_2^2\frac{1}{l^3}8\pi^4(1+16y^2) \ldots\ldots (40b)$$

and

$$\int_{l/2}^{l} \beta^2 (l-z)^2\, dz = a^2{}_2\, l^3 \frac{1}{2}\left(\frac{1}{24}\, y^2 - \frac{5}{64\,\pi^2}\, y^2 - \frac{4}{9\,\pi^2}\, y + \frac{1}{24} - \frac{1}{16\,\pi^2}\right) \ldots (40c)$$

which values, in Eq. 38, yield:

$$P = \frac{E\, I_y\, d}{l^3}\, 2\,\pi \sqrt{\frac{\frac{l^2}{a^2}(1+4\,y^2) + 4\,\pi^2 (1+16\,y^2)}{2\left(\frac{1}{24}\, y^2 - \frac{5}{64\,\pi^2}\, y^2 - \frac{4}{9\,\pi^2}\, y + \frac{1}{24} - \frac{1}{16\,\pi^2}\right)}} \ldots (41)$$

and, therefore,

$$f = \frac{I_y\, d^2}{I_x\, a^2}\, E\, \frac{a^2}{l^2}\, \frac{\pi}{4} \sqrt{\frac{\frac{l^2}{a^2}(1+4\,y^2) + 4\,\pi^2 (1+16\,y^2)}{2\left(\frac{1}{24}\, y^2 - \frac{5}{64\,\pi^2}\, y^2 - \frac{4}{9\,\pi^2}\, y + \frac{1}{24} - \frac{1}{16\,\pi^2}\right)}} \ldots (42)$$

To express the unit stress, f, in terms of the unsupported length of the beam instead of the full length of the beam, as in Eq. 42, l in Eq. 42 should be multiplied by 2. The value of f is then expressed by Eq. 4 in which

$$k = \frac{\pi}{8} \sqrt{\frac{\frac{l^2}{a^2}(1+4\,y^2) + \pi^2 (1+16\,y^2)}{2\left(\frac{1}{24}\, y^2 - \frac{5}{64\,\pi^2}\, y^2 - \frac{4}{9\,\pi^2}\, y + \frac{1}{24} - \frac{1}{16\,\pi^2}\right)}} \ldots\ldots (43)$$

in which l equals the unsupported, or half, length of beam. The critical values of y and of k as determined by Eq. 43 have been computed and are tabulated in Tables 7(*c*) and 1(*c*).

Two-Bay Beams; Uniformly Distributed Load

In the case of a uniformly loaded beam with lateral restraint at the middle cross section (Fig. 8), by the method previously used for the laterally unsupported beam, an equation similar to Eq. 28 can be written:

$$\frac{w^2}{8\, E\, I_y} \int_0^l \beta^2 z^2 (l-z)^2\, dz \pm \frac{w\, d}{4} \int_0^l \beta^2\, dz - \frac{E\, I_y}{8} \frac{l^2{}_x d^2}{a^2\, l^2} \int_0^l \left(\frac{d\beta}{dz}\right)^2 dz - \frac{E\, I_y}{8}\, d^2 \int_0^l \left(\frac{d^2\beta}{dz^2}\right)^2 dz = 0 \ldots (44)$$

Eq. 44 may be solved for the loading $w\, l$:

$$w\, l = E\, I_y\, dl \left\{ \mp \int_0^l \beta^2\, dz + \sqrt{\int_0^l (\beta^2\, dz)^2 + \int_0^l \beta^2 z^2 (l-z)^2\, dz \left[\frac{l^2}{a^2}\frac{1}{l^2} \int_0^l \left(\frac{d\beta}{dz}\right)^2 dz + \int_0^l \left(\frac{d^2\beta}{dz^2}\right)^2 dz\right]} \right\} \div \left[\int_0^l \beta^2 z^2 (l-z)^2\, dz\right] \ldots (45)$$

With solutions for integrals, not solved before:

$$\int_0^l \beta^2\, dz = a^2{}_2\, l\, \tfrac{1}{2}\,(1 + y^2) \ldots\ldots\ldots\ldots\ldots\ldots (46a)$$

and

$$\int_0^l \beta^2 z^2 (l - z)^2\, dz = a^2{}_2\, l^5 \frac{1}{8}\left(\frac{2}{15} y^2 - \frac{1}{2\,\pi^2} y^2 + \frac{63}{128\,\pi^4} y^2 - \frac{320}{27\,\pi^4} y + \frac{2}{15} + \frac{3}{8\,\pi^4}\right) \ldots . (46b)$$

Eq. 45 becomes

$$w\, l = \frac{4\, E\, I_y\, d}{l^3}\left\{ \mp (1 + y^2) + \sqrt{(1 + y^2)^2 + \pi^2 \left[\frac{2\, y^2}{15} - \frac{y^2}{2\,\pi^2} + \frac{63\, y^2}{128\,\pi^4} - \frac{320\, y}{27\,\pi^4} + \frac{2}{15} + \frac{3}{8\,\pi^4}\right] \times \left[\frac{l^2}{a^2}(1 + 4\, y^2) + 4\,\pi^2 (1 + 16\, y^2)\right]}\right\} \div \left[\frac{2\, y^2}{15} - \frac{y^2}{2\,\pi^2} + \frac{63\, y^2}{128\,\pi^4} - \frac{320\, y}{27\,\pi^4} + \frac{2}{15} + \frac{3}{8\,\pi^4}\right] . . (47)$$

and, with—

$$A = \frac{2}{15} y^2 - \frac{1}{2\,\pi^2} y^2 + \frac{63}{128\,\pi^4} y^2 - \frac{320}{27\,\pi^4} y + \frac{2}{15} + \frac{3}{8\,\pi^4} \ldots\ldots (48)$$

—the formula for stress becomes:

$$f = \frac{I_y\, d^2}{I_x\, a^2} E \frac{a^2}{l^2} \frac{1}{4\, A}\left\{ \mp (1 + y^2) + \sqrt{(1 + y^2)^2 + \pi^2 A \left[\frac{l^2}{a^2}(1 + 4\, y^2) + 4\,\pi^2 (1 + 16\, y^2)\right]}\right\} \ldots . (49)$$

If each l in Eq. 49 is multiplied by 2, to express the unit stress, f, in terms of the unsupported length instead of the full length of the beam, as was done for the concentrated load (Eq. 42), then the value of f for uniformly distributed load is expressed by Eq. 4 in which, with the load at the top flange,

$$k = \frac{1}{16\, A}\left\{ - (1 + y^2) + \sqrt{(1 + y^2)^2 + 4\,\pi^2 A \left[\frac{l^2}{a^2}(1 + 4\, y^2) + \pi^2 (1 + 16\, y^2)\right]}\right\} \ldots (50a)$$

at the centroid,

$$k = \frac{\pi}{8A}\sqrt{A\left[\frac{l^2}{a^2}(1+4y^2)+\pi^2(1+16y^2)\right]} \quad \ldots\ldots\ldots (50b)$$

and, at the bottom flange,

$$k = \frac{1}{16A}\left\{(1+y^2) + \sqrt{(1+y^2)^2+4\pi^2 A\left[\frac{l^2}{a^2}(1+4y^2)+\pi^2(1+16y^2)\right]}\right\} \ldots (50c)$$

The critical values of y and the corresponding values of k have been computed and are given in Tables 7(*c*) and 1(*c*).

APPENDIX II—NOTATION

The following letter symbols, used in the paper and in its discussions, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932. As nearly as possible, consistent with standard practice, they also follow the nomenclature used by Professor Timoshenko:[2]

A = a substitution factor; for simple beams with a concentrated load at the center, A is defined by Eq. 19; for simple beams with a uniformly distributed load, A is defined by Eq. 34; and, for a two-bay beam with a uniformly distributed load, A is defined by Eq. 48;
a = a torsional bending constant; also where defined, a_1, a_2, a_3, a_4 = maximum ordinates of sine curves (Eqs. 14 and 39);
B = transverse flexural rigidity of a beam = $E I_y$;
b = width of compression flange of beam or girder;
C = torsional rigidity of beam = $E I_y d^2/(4a^2)$;
D = diameter of an inscribed circle in a flange section;
d = depth of a beam or girder:
  d' = effective depth of a plate girder (see Fig. 6);
  d_c = depth of a plate girder between flange centroids;
E = modulus of elasticity;
f = unit stress;
I = moment of inertia about an axis denoted by an appropriate subscript;
K = a torsion constant;

k = a substitution factor; for concentrated loads on a simple beam, k is defined by Eqs. 21; for uniformly distributed loads on a simple beam, k is defined by Eqs. 36; for a concentrated load on a two-bay beam k is defined by Eq. 43; and, for uniformly distributed loads on a two-bay beam, k is defined by Eqs. 50;

l = length; in general, l denotes "unsupported length of the compression flange," with exceptions as described in the text;

M = moment, with subscripts indicating the plane of reference;

P = a concentrated load;

r = radius; r_f = fillet radius;

s = length along any arc; ds = an increment of s, with appropriate subscript;

t = thickness of compression flange:

t' = effective average thickness of cover plates (Eq. 14);

t(max) = maximum cover-plate thickness;

t(min) = minimum cover-plate thickness;

u = deflection of any element;

V = work with subscripts as defined in the text;

w = load per unit length of span;

x = a coordinate;

y = a coordinate; also, where defined, $y = \frac{a_3}{a_1}$ and $y = \frac{a_4}{a_2}$;

z = a coordinate;

β = angular deflection of a beam at any point; β_o = angle of twist of the beam at its center line;

ζ = a coordinate in the (ξ, ζ, η)-system;

η = a coordinate in the (ξ, ζ, η)-system; and

ξ = a coordinate in the (ξ, ζ, η)-system.

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DISCUSSIONS

BALANCED DESIGN IN URBAN TRIANGULATION

Discussion

BY HENRY W. HEMPLE, L. G. SIMMONS, C. A. WHITTEN, AND RONALD F. SCOTT

HENRY W. HEMPLE,[8] M. AM. SOC. C. E.[8a]—The subject of urban triangulation design has been presented by Mr. Hopkins in a vivid manner. Engineers engaged on this type of work will benefit by his general concept of design. The principles which the author recognizes have been in use for many years and are established as standard for precise surveys.

The comparison of triangulation design to that of structures is an apt illustration, but the comparison is analogous only to a certain point; beyond that the similarity does not exist. It should be understood, however, that specifications for design, observational procedure, computation, and adjustment are so rigid for urban and precise triangulation surveys that, if the basic measurements and observations were adequate and if the specifications were otherwise adhered to, there would be no weak determination of positions or lengths in the scheme.

In the design of structures there are various known factors, or those that may be correctly estimated for, producing stresses for which structural members whose strength characteristics are known can be provided. In triangulation work, unknown physical conditions may be of a nature to produce lateral refraction of light rays and introduce errors in the directional measurements. These effects cannot be called errors due to structural failures, and there is no element of design that can be introduced to guard against them. They are errors resulting from unknown causes which cannot be compensated for, and only by reobservation under good atmospheric conditions can they be eliminated.

If the computation and adjustment of a triangulation scheme have met the specifications, these discrepancies must be due to imperfect measurements or observations. The degree of failure that might be acceptable can be antici-

NOTE.—This paper by Charles D. Hopkins was published in November, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1946, by Walter S. Dix.

[8] Chf., Div. of Geodesy, U. S. Coast and Geodetic Survey, Washington, D. C.

[8a] Received May 10, 1946.

pated from the strength of any design. If the failure exceeds this limit, certainly the design is not at fault. In this concept, Mr. Hopkins' statement (see heading, "Introduction") that "Localized error that cannot be corrected properly can rightly be called structural failure" will have no basis.

Many factors enter into the formation of a scheme of triangulation. Topographic and other conditions will not always permit the location of the stations at points that will give the ideal design. Lines over which undue refraction may be expected must be avoided. Stations on opposite ends of the lines of the scheme must be tested for intervisibility. Base lines must be provided at the proper intervals so that adequate length control is available. Connections to adjacent federal control must be made so that there will be a continuous coordinated scheme throughout the area. Adequate strength of figures must be inherent in the triangulation figures formed. The scheme selected should be such that it can be carried through with economy and yet maintain the accuracy desired.

One is not justified in spending an undue amount of effort and time with the resulting increased cost, striving for ideal figures, when there are other possible solutions easier of execution which will satisfy the criteria for urban surveys. The competent engineer evaluates all these considerations and arrives at a scheme that can be executed economically and still satisfy the requirements of the specifications.

The U. S. Coast and Geodetic Survey has adopted the practice of providing at least one base line near the larger cities for length control of the federal net in that locality. Lines of the federal net and their azimuths may be accepted into the urban scheme with assurance that they are determined with adequate accuracy for city survey purposes. In cities where urban surveys exist at the time the federal net is carried to them, connections are made to the local surveys and their lengths are held if the work has been done with standard specifications for that type of control.

Mr. Hopkins has considered only the internal forces affecting a scheme of triangulation. When area triangulation is adjusted to an established scheme surrounding it, the maximum external forces are encountered. The engineer can be certain that, if the observational data will withstand the test of this adjustment, the fundamental control and the new survey have each been determined with high accuracy.

Mr. Hopkins has shown a keen appreciation for strong figures in triangulation and this fundamental principle must be recognized by all those engaged in urban surveys.

L. G. SIMMONS,[9] ESQ.[9a]—It is agreed that well-shaped figures with double determination in length are essential to high precision in geodetic triangulation. The writer does not agree on the importance that Mr. Hopkins attaches to the effects caused by the variations in his sample figure (Fig. 1). These effects are so small that they are likely to be completely obscured by more prominent effects arising from conditions entirely independent of figure design. It is

[9] Prin. Geodetic Engr., U. S. Coast & Geodetic Survey, Washington, D. C.

[9a] Received May 23, 1946.

quite possible that the writer has missed the point, but to him it seems that Mr. Hopkins' reasoning does not follow a logical course.

For example, he begins with a symmetrical central-point hexagon and then assumes a set of observations closely approximating Fig. 1. As soon as he has assumed this set of observations, the perfect hexagon figure no longer has a place in the problem. It is nothing more than something he had in his mind before the analysis was started. The set of assumed observations is then adjusted by the method of least squares and the results tabulated as Δ-values or differences between the adjusted directions and the directions defined by the angles of the perfect hexagon. The significance of these Δ-values escapes the writer. There is no reason that the adjusted values of the angles approach those of a perfect central-point hexagon other than the fact that the assumed values approximated this figure.

Even if these Δ-values do have significance, it is not clear to the writer why the merit of the figure design should be judged by the closeness with which plotted values cling to a line drawn between the high and low values. On this basis, why does Mr. Hopkins state that stage 3, where complete double determination is first reached, is the most efficient figure design? Is it because the Δ-values, which have no meaning in the first place, occur in a uniformly ascending series of values? To the writer it is more important to keep the Δ-values low (if they do have a meaning), and on this basis it would seem that stage 0 is more efficient than stage 3. Moreover, the differences among all stages appear of small consequence, even if the assumed triangle closures were doubled.

As a practical proof of what figure design will do toward securing length agreement between measured bases, Mr. Hopkins cites several schemes of urban triangulation in which (in most cases) length closures were considerably better than 1 part in 1,000,000. The writer maintains that any length closures that are much better than 1 part in 200,000 are purely accidental and extra precision as implied by better figures is more apparent than real. The reason for this statement is twofold: First, lengths of the best standardized base tapes, with an accuracy of 1:200,000, are scarcely known. Certainly there are none more accurate than 1:500,000; and, second, the effects of systematic errors, which are present particularly in urban areas, preclude any certainty of a much better length closure. No amount of extra refinement in figure design will overcome the limitations already imposed by observing conditions (uncertainties in lateral refraction), and the limitations imposed by the ever-present systematic errors in triangulation work, too numerous to mention.

Of course, strong figures should be used and single triangles avoided where the highest precision is sought, but Mr. Hopkins is putting into figure design a refined value which cannot be extracted economically—if at all.

C. A. WHITTEN,[10] ESQ.[10a]—The selection of the type of mathematical analysis of a triangulation scheme is frequently dependent on the desires of the individual making the particular study. The possible types of investigations with all the variations which naturally develop present a problem for

[10] Chf., Section of Triangulation, Div. of Geodesy, U. S. Coast & Geodetic Survey, Washington, D. C

[10a] Received May 23, 1946.

which there would be no end. When only one type of investigation or procedure is used, it must be established that the methods employed are rigid at least in principle.

A new method of analysis for a study of triangulation design is presented by Mr. Hopkins. In general, the results derived are not contradictory to established fact. Some of the author's conclusions are based no doubt on his experience, plus the "exercise of logic," rather than on the findings from a standard mathematical treatment. It is because of this that it seems necessary to caution those who would pursue this problem in further variations.

At the outset, the method of adjustment, whether by the angle or the direction method, should be established definitely. Both cannot be used simultaneously or interchangeably as Mr. Hopkins has done. Having established the method of adjustment to be used, it is permissible to make some assumptions. However, the writer questions the procedure of assuming a rigid figure and then assuming a set of observational errors. What will hold the true figure rigid? There could be a large number of mathematically consistent or rigid figures within the range of the assumed errors without creating an error greater than any of the original assumed errors. Thus, this method does not seem feasible for a general mathematical development. Even the distribution of the assumed errors was made in a systematic manner rather than according to probability. Why, then, use least squares? .

Another unusual treatment is the development and tabulation of the Δ-terms. Since the assumed observational data are not truly observational, according to the laws of distribution, the graphs of the Δ-terms do not furnish conclusive evidence concerning the various stages. The ideal adjustment would be the one in which the Δ-values are small, and Mr. Hopkins has probably selected the wrong area of the graph for determining the "efficiency index." The mean Δ-term is a more logical value for an index.

The results obtained from the calculations of the seven stages of design should not be compared with each other without giving consideration to the mean of the assumed errors which were distributed by pattern. These mean errors are:

Stage of design	Mean error of angle (seconds)
0	0.067
1	0.088
2	0.097
3	0.113
4	0.122
5	0.126
6	0.127

Since there is a large change in the mean errors of the various stages, similar changes will be found in the probable errors and average corrections. However, the author has attributed the changes in probable errors and average corrections to the stages of design rather than to the magnitude of errors in the different figures. The mean errors have not been balanced sufficiently for a satisfactory comparison of the various stages.

Mr. Hopkins recognizes many of the difficulties that are encountered in determining the accuracy of any triangulation net or figure. When rigid specifications are used to limit the magnitude of errors of observations, the least squares method of adjustment will produce very satisfactory results. From a least squares adjustment, it is possible to determine the effect of an error of any one observation on the other lines of the scheme. Such a process as this, when applied to all lines of a scheme through various stages of development, would be very laborious and the value of the results obtained might not be worth the effort. However, such a method would produce results that could be interpreted with certainty since the procedure is rigid and conforms with accepted mathematical practice.

RONALD F. SCOTT,[11] JUN. AM. SOC. C. E.[11a]—It is hoped that this paper is the beginning of a new and wholesome approach to the problem of covering an area with triangulation of a more or less predictable accuracy. To this writer's knowledge all too little attention has been given to this very important phase of precise control surveys. The design of a triangulation scheme to provide uniformly precise control over a large area has been too much a matter of guesswork; or, perhaps, it has been classed as an art rather than a science. Much work has been done in adapting triangulation to the problem of a chain of quadrilaterals establishing control over long distances; but its use for an area has been insufficiently explored.

Those who have undertaken the design of a scheme for an area have too often been regarded as individuals apart from normal human beings with a special sense of things mathematical and geometrical. It will be very important to remove this phase of surveying from that highly specialized classification into the realm of things more common. Much work has been done to make the little-known facts of precise surveying available to a broader group of engineers. However, the profession is still a long way from providing the necessary and desirable tools to complete the numerous, carefully executed control surveys that will be required before the various systems of state-wide plane coordinates can come into general use. In the past decade some engineers have declined to undertake the supervision of precise-control projects involving triangulation simply because they have felt that the problems and techniques were far too involved for someone with ordinary engineering training. This concept of precise surveying can be done away with only by the development of design methods that will be suitable for widespread use.

The author's brief mention of a system of templets which will illustrate, graphically, the structural integrity in a triangulation scheme is the kind of thing which could well be expanded further. If engineers are required to take up this work they should be assisted to the greatest possible understanding of the significant factors, not only by mathematical reasoning but by utilization of all possible graphical and mechanical means. This relationship of geometric figures to one another in a triangulation scheme is the phase of the work that is the most difficult to understand. It is so intangible that it may too easily escape proper attention if, indeed, it receives any attention.

[11] Director, Upper East Tennessee Office, Tennessee State Planning Comm., Johnson City, Tenn.
[11a] Received May 27, 1946.

The hopelessly disorganized status of cadastral surveys, particularly in some parts of the eastern United States, will eventually bring about an insistent demand for sweeping reforms in surveying methods. It will be necessary for the engineering profession to have on hand, ready for use, the necessary criteria and design procedures in sufficient detail to enable the design of control surveys with the same assurance of success as can be had in the design of structures. Unfortunately, the advantages of sound surveying practice are apparent only to those who are most intimately connected with the very worst of the difficulties that arise from a short-sighted policy on control requirements. Even today a very few engineers, not to mention the laymen, realize the advantages and economy of a properly developed system of dimensional control for areas of extensive physical development.

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DISCUSSIONS

UPLIFT PRESSURE IN AND BENEATH DAMS
A SYMPOSIUM

Discussion

BY GORDON V. RICHARDS

GORDON V. RICHARDS,[52] JUN. ASCE.[52a]—The impressive thing about the work reported by Mr. Selim is the extreme simplicity of the method as compared to the complexity of the problem. It requires no great amount of equipment and could be used for problems far more complicated than those presented.

One is led to wonder if it would be possible to find the uplift pressures under a dam where drains were installed. It seems likely that this could be done. As with the models in the paper, the stream-bed pressure upstream from the dam would be represented by a plate maintained at, say, 10 v. The downstream plate could be maintained at zero potential. Between these two (that is, under the foundation of the dam) a third conductor, with a size and shape depending on the type of drainage installed in the dam, might be maintained at some intermediate potential depending on the efficiency of the drainage under the prototype.

Possibly, also, the method could be applied without undue difficulty to three-dimensional problems of laminar flow. The only requirements to be met are that the model and prototype be geometrically similar and that the boundary pressures existing in the prototype be duplicated in the model as electric potential. In the case of a three-dimensional model it would be necessary to measure the potential at a point, instead of at a vertical line as is allowed in the two-dimensional problem. Perhaps this could be accomplished by using, for the probe, a well-insulated wire with an exposed copper sphere (a sixteenth or an eighth of an inch in diameter) on the end. With the proper measuring instruments, a three-dimensional model might be explored with the copper sphere. One difficulty with this idea will be the construction of the model in such a way that the liquid is maintained in the right shape. For a dam in a steep ravine, for example, the best method might be to invert the model so that the bottom of the tank containing the liquid represents the ground surface inverted.

NOTE.—This Symposium was published in December, 1945, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: June, 1946, by W. H. Holmes, W. A. Perkins, and S. P. Wing.

[52] Instr., Mech. Eng. Dept., Univ. of California, Berkeley, Calif.

[52a] Received May 21, 1946.

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DISCUSSIONS

NEW PROJECT FOR STABILIZING AND DEEPENING LOWER MISSISSIPPI RIVER

Discussion

BY CHARLES W. OKEY, AND F. NEWHOUSE

CHARLES W. OKEY,[2] M. ASCE[2a]—In describing the new project for stabilizing and deepening the Mississippi River, and recording a condensed history of the development of the present flood control plans, Mr. Senour has performed a service of great value. This paper is a most valuable summary of the results of geologic investigations by the Corps of Engineers, U. S. Army, for the past several years on the Mississippi River below Cairo, Ill.

This new project is gigantic, as compared to anything tried elsewhere, both in the cost of the proposed work, and the size of the river and volume of flows involved. The field observations and the model investigations give good reason to believe that the principal objects of the project will be attained in the years to come. The principal uncertainty seems to be: What will be the progressive development of the crossing bars between the bends? If the supply of bed load from the upstream caving banks were reduced by revetment, then dredging might be reduced. As Mr. Senour states, if the migrating bends have their position and curvature fixed by revetment, low dikes would be a possible solution for a channel across those bars where the required depth is not maintained by the river itself. Without such low dikes to confine the low-water flow to a predetermined course, the elevation of the crossing bar might be reduced by less bed-load supply, but with a corresponding reduction in the elevation of the water across the bar, resulting in no substantial increase in navigable depth.

The prevention of the loss of levees on caving banks and the prevention of a gradual increase in the length of the channel, so beneficially reduced by cutoffs in the past few years, are objectives of great importance. They are much to be desired even though a great increase in the navigable depth over the crossing bars might not be automatically attained and continuously preserved at low-water stages.

NOTE.—This paper by Charles Senour was published in February, 1946, *Proceedings*.

[2] Head Hydr. Engr., TVA, Knoxville, Tenn.

[2a] Received April 4, 1946.

The writer has been interested in this matter since March, 1911, when he read a manuscript copy of a paper by the late Maj. T. G. Dabney, for many years the chief engineer of the Yazoo-Mississippi Levee District. That paper (which was not published) had been originally written as a plea to the Mississippi River Commission to do what is now proposed by the project authorized in the 1944 Act. In reading Mr. Senour's paper, the writer's mind traveled along a familiar path which seemed to agree with Major Dabney's plea to the Mississippi River Commission. This plea of Major Dabney's if found in the records of the Mississippi River Commission or in the levee board office at Clarksdale, Miss., would constitute a most interesting commentary on Mr. Senour's paper.

F. NEWHOUSE,[3] ESQ.[3a]—The practice of building bigger and better levees to control a meandering river has proved ineffective on the Lower Mississippi, as is revealed in this valuable and interesting paper. The experiments and research described in the paper appear to have led the authorities to decide to deal with this engineering problem by adopting a method whose necessity can be deduced from basic theory.

Any river carves its course through its own alluvial plane by the erosive power of its current. This erosive power depends on the velocity of the water, and the degree of erosion depends on the resistance of the soil. A river, therefore, achieves stability when its velocity is such that it cannot erode its channel. Velocity, of course, depends on slope. This fact leads to the basic theory that the shape of the channel of a river depends on two things only—(1) the slope and (2) the soil through which the river flows. The slope in turn depends on two quantities—(*a*) the fall from the source to the sink and (*b*) the length of the course.

For practical purposes, the fall must be accepted as fixed over any river as a whole, since it can only be changed to a significant extent by earth movements. On individual reaches the slope could be altered by building weirs and falls, but such construction would be attended by economic and engineering difficulties, probably prohibitive because of their extent on large rivers.

The other element in slope, length of course, is altered by the forces of nature in two ways: The river eats into one bank or the other forming a curve and carries the eroded material onward, in a series of repeated depositions and erosions, until it reaches the sea. Shoals are built up and the land gradually advanced, by the deposition of the eroded material and of the silt brought down from the catchment area by normal (or abnormal) denudation. At some river mouths this formation is very conspicuous, as, for instance, at the Rosetta and Damietta mouths of the Nile River. Any map, even a small-scale map, shows the capes formed in the smooth coast line of Egypt.

The combined effects of the curves formed by erosion and of the deposition at the mouth lengthen the course of a river and therefore reduce the slope and hence the velocity. Finally, a state of equilibrium is established in the flood bed of the river, when the velocity is so reduced that it can no longer produce

[3] Twickenham, Middlesex, England.
[3a] Received May 31, 1946.

scour. Such a condition is easily upset by slight local variations in velocity whose effect is to set the deep channel of the river swinging from side to side. The channel then becomes a series of curves, full of shifting shoals, and tends constantly to erode one bank or the other. Everything possible has been done by nature in the way of reducing slope and velocity to fit the soil, without producing a satisfactory river channel for man's purposes. Therefore, action must be taken on the second factor that affects the course of a river—namely, the soil through which the river flows.

Nature changes soil by denudation but not in a manner designed to offer greater resistance to erosion. Excessive denudation of the catchment area causes floods to carry a heavier charge of silt, and produces sandbanks, inequalities in current, erosion, and general instability; but it does not improve a river—just the reverse! In fact, soil cannot be improved by nature, from the point of view of resistance to erosion, and the indisputable conclusion is that it must be protected by the art of the engineer—by the use of stone. Thus theory has led to the conclusion that revetment alone can stabilize the course of a river.

Cutting off a bend on a river must increase velocities locally, and start again the aforementioned erosion and other ill effects. If every cutoff is not revetted as it is built, the "Works" should be given the code name of "Operation Sisyphus."

It has been decided to revet cutoffs on the Mississippi, but the extent of the revetment of the cross section has not been stated in the paper. If the banks alone are revetted, there will be erosion of the bed and the revetment on the slopes will be underscoured, and will collapse, with consequences easy to foresee. It would be of interest to know the steps being taken to meet this danger and, eventually, how they succeed.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ENGINEERING EDUCATION

PROGRESS REPORTS OF THE SOCIETY'S PROFESSIONAL COMMITTEE ON ENGINEERING EDUCATION, FOR 1944 AND 1945

Discussion

BY LOWELL O. STEWART, AND ROY M. GREEN

LOWELL O. STEWART,[3] M. ASCE.[3a]—Useful data are presented in the 1944 and 1945 reports of this committee. Each educator will welcome the tables of the 1944 report which show the percentages of total curriculum time devoted to the various subjects that make up the curriculum. He will also welcome the committee's suggested time distribution of these subjects in a typical civil engineering curriculum, because he will be able to compare his curriculum with others, and with the suggested composite.

The data in the tables of the 1945 report supply basic information that will be helpful in determining which courses shall be included in the typical civil engineering curriculum of the future. It is hoped that the committee will analyze these data, or gather additional data, to the point where it can state what civil engineering courses, and to what extent each, should be included in a typical civil engineering curriculum. It is the writer's opinion that certain courses of specified minimum content, representing the recognized fields in civil engineering, such as surveying, highways, structures, sewerage and water supply, should be included in every civil engineering curriculum. This means that the 30% of civil engineering courses recommended by the committee would be divided into the specific courses that the committee believes should be studied by every student who is pursuing a civil engineering curriculum. This kind of subdivision is as important as that which determines the percentage of the student's total time that should be given to the "subject groups." The data for use in preparing this proposed distribution of civil engineering subjects are shown in Tables 3(*d*) and 4(*b*). This listing of civil engineering subjects (with some supplemental details) could serve as the basis for a tabulation that

NOTE.—These reports were published in March, 1946, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: June, 1946, by Frederic Bass, and John B. Wilbur.

[3] Prof. and Head, Dept. of Civ. Eng., Iowa State College, Ames, Iowa.

[3a] Received May 6, 1946.

would specify the breadth, or depth, or upper range of each subject. For example, the proposed tabulation of civil engineering subjects for a civil engineering curriculum might be made in part as follows: Surveying, including mapping, photogrammetry, and curves; sanitary, including hydrology and water supply sewerage; and structures, including foundations, statically determinate and indeterminate theory, and design in steel, reinforced concrete, and wood.

Specific information of this nature is necessary if understanding and agreement are to be reached by those who are interested in and responsible for planning the civil engineering curricula. Such specificity will leave ample subject matter area within which a department or professor can exercise its, or his, individuality. At the same time, it will help to establish the elements that should be in every civil engineering curriculum. The entire curriculum should be built in this manner. Then, after all the necessary courses have been listed, a total should be reached which would indicate the optimum length of the curriculum in terms of months or years.

An analysis of the data in the committee's 1945 report will indicate a need for a lengthened (5-year) curriculum. A summation of the credits necessary for the "Other Subject" courses (rated as of "great importance" and somewhere between "moderate importance and great importance") might easily make a total that exceeds the 20% recommended for the humanistic-social stem. Furthermore, the answers to questions (2) and (3), which refer to the desirability of preferring nontechnical courses to specialized engineering courses, show a large majority for greater emphasis on nontechnical courses and less emphasis on specialized engineering courses. Opinion was evenly divided on the question of whether such a change in emphasis would have an unfavorable effect upon the professional competence of the civil engineer. The implication of these answers points to the need for a curriculum that is longer than four years if engineering educators are to place more emphasis on nontechnical courses and maintain the professional competence of the civil engineer at the present standard.

Perhaps the committee will make some further analyses of the data and seek answers to questions regarding the desirability of a curriculum longer than four years. There is nothing sacred about the four-year curriculum. Other professions, such as medicine, veterinary medicine, and law, have extended their curricula beyond four years. Civil engineering educators and civil engineers are agreed that civil engineers (and all other engineers) need more humanistic-social education. The amount needed seems to be 20% of the curriculum. To provide space for this material the committee (1944 report) recommends "some reduction or replacement of time allocations to presently taught subjects." In the 1945 report, the tabulation from the 1944 report is repeated with the conclusion: That the 1944 subdivision of time is reinforced by the 1945 returns, and that it is possible to devote 20% to nontechnical subjects and yet to include the fundamental technical and science content. It is suggested that the committee indicate how the 1945 data show this correlation with the 1944 tabulation which is an average of civil engineering curricula.

The phrase "humanistic-social courses" which is used in the tabulation, and has been current for several years in the literature of engineering education,

needs defining. A better designation for the courses in this area would be "general education," which the American Council on Education defines as "those phases of nonspecialized and nonvocational education that should be the common possession, the common denominator, so to speak, of educated persons as individuals and as citizens in a free society." Above all, there must be a clear statement, and understanding by everyone, of the objectives of general education. When that has been done, engineering educators may proceed with the selection and organization of the courses that will attain those objectives. Furthermore, they will be able to make effective use of the "general education" content of technical courses.

In Table 3 (in which is rated the effectiveness of the civil engineering graduates of the past decade), the committee has selected many of the important traits that the successful engineer must have. Most of these apply equally to successful men in every profession or occupation. Doubtless the committee is justified in its belief "that the replies * * * of the questionnarie * * * constitute in some degree an indictment of engineering teaching." One of the critical deficiencies of professional men and specialists is their inability to do well what the Harvard report, "General Education in a Free Society,"[4] states is the goal of general education. The traits and characteristics of mind which, the report says, should be fostered above all others are: (1) To think effectively, (2) to communicate thought, (3) to make relevant judgments, and (4) to discriminate among values. Each teacher does his share of the job of developing these traits in so far as the narrow limitations of his own course apply; but there is poor integration of these, and other traits, among courses. Oftentimes a curriculum, which should be an integrated group of courses, is a collection of courses in which the subject matter and objectives are not correlated. Here is an area where the committee, in cooperation with the Society for the Promotion of Engineering Education (SPEE) Committee, may do some effective work.

It is of interest to learn that the committee finds in the civil engineering graduates of the past decade many of the deficiencies that were found in the past decade by the Wickenden report.[5] Although these deficiencies are prevalent in all professions and occupations, they seem particularly notorious to the engineer because he demands, and usually gets, exactness and correctness in his technical work. In general, this lack of effectiveness in such attributes as "ability to formulate a formula for themselves," "ability to undertsand the limitation of formulas," and "ability to consider all cost factors" may be traced to insufficient ability to make relevant judgments and to discriminate among values, which the Harvard report says are two of the main goals of general education. Four possible routes by which the problem may be attacked are: (1) Develop a broader curriculum (one reaching more areas of knowledge), (2) provide better teachers (probably with broader educations), (3) improve teaching methods so that the student is called upon to make effective use continuously of the things that he has learned, and (4) continue the engineer's education after graduation.

[4] "General Education in a Free Society," Rept. of the Harvard Committee, Harvard Univ. Press, Cambridge, Mass., 1945.

[5] "Report of the Investigation of Engineering Education, 1923–1929," S.P.E.E., Pittsburgh, Pa., 1930.

ROY M. GREEN,[6] M. ASCE.[6a]—As a member of the cooperative committee in Civil Engineering Education of the Society for the Promotion of Engineering Education (S.P.E.E.) it would seem advisable that the writer present a brief discussion of the 1945 report on "Engineering Education" published by the ASCE. The comments expressed, however, should be considered as the thoughts of the writer and not necessarily the opinion of the members of the S.P.E.E. cooperative committee.

In advocating the use of a questionnaire, the cooperative committee assumed that ASCE was the professional society that represents civil engineering at its best, that its membership was composed of a fair cross section of the profession, and that it included a large number of men intensely interested in civil engineering education. The considered opinion of such a group was believed to be of great significance if the questions asked could be answered competently. It was decided to interrogate the membership of the Society only on matters relating to the education of all civil engineering students. The questions sought opinions on (*a*) the qualities of the recent graduates, or the effectiveness of the educational program of the past few years; and (*b*) the educational materials or courses considered as important in the development of all civil engineers.

Since it was realized that the opinion of any member of the Society would be materially influenced by his own personal experiences, an effort was made to record a meager background of information which might influence the opinion of the member reporting. It was believed that the expressions of opinion collected by the questionnaire might serve as primary guides to those who were attempting a revision of civil engineering curricula at this time.

Civil engineers are certainly indebted to the ASCE committee and the Board of Direction for their thoughtfulness in publishing the report and the summary of the replies to the specific questions. Little fault should be found with most of the general conclusions stated in that report.

It is very unfortunate, however, that a system of tabulation was used which did not subject the results to some checking process. The report states that 1,035 questionnaires were completed and returned. Table 4 indicates that 1,036 persons checked "differential and integral calculus"; 1,051 checked "materials testing laboratory"; and 1,113 checked "irrigation and drainage"; 1,046 checked "sewerage and sewage disposal"; 1,035 checked "English composition"; and 1,132 checked "public speaking." It is obvious that there must have been more than 1,035 questionnaries returned or there are errors in the tabulation. Later in the report it was noted that 1,080 questionnaires were returned. The fact that there are errors in tabulation, therefore, is obvious.

Since the results constitute an accumulation of significant opinion it is hoped that, in any subsequent report, a revised tabulation may be made which will include a tally of those who did not express an opinion on each query. If this is done, the data can be used with greater confidence by those attempting to frame curricula.

The answers to the questionnaire as recorded are only a composite of the opinions of a large group of informed men who are vitally and sincerely inter-

[6] Dean of Eng., Univ. of Nebraska, Lincoln, Nebr.
[6a] Received July 1, 1946.

ested in civil engineering education. Since each reply is influenced by the personal experience of the individual who marked the questionnaire, maximum value of the results can be obtained only by dividing the replies into groups to determine whether Juniors have different opinions than Members; whether the opinion of the men whose duties are largely technical varies from that of those in administrative positions; or whether the opinion of those interested in "airports" varies from that of those specializing in "highways." If there are differences, what are they? What is their explanation? If the results are subdivided they may become of extreme value to many committees of the Civil Engineering Division of S.P.E.E. As recorded in the report, the data are extremely interesting but not as constructive as they might become.

Should an attempt be made to draft a "model" curriculum? Results obtained by the educational process of curriculum study and teaching effort are wholly individual products. Education is only of value when it has resulted in the stimulation of the student to such a degree that ultimate professional achievement is made a strong probability through the experience of the student working with subject field and teacher. Teaching staffs are made up of varying types of individuals. If the curriculum does not fit the teaching staff, optimum results cannot be obtained. It is hoped that the various civil engineering departments will not attempt to match their curricula too closely but rather that each will preserve its individuality and develop its natural tendencies—even though certain basic knowledge is obviously necessary for all civil engineering graduates.

What about nontechnical courses? Of the persons returning the questionnaire, 11% did not reply to query (2) (Fig. 3)—

> "If colleges place more emphasis on nontechnical courses and less emphasis than formerly on specialized engineering courses, would the effect upon the influence and prestige of civil engineers be favorable or unfavorable?"

Of those who answered 74% believed that the effect upon the influence and prestige would be favorable and only 26% that it would be unfavorable. The opinion seems positive. To query (3) (Fig. 3)—

> "If the colleges place more emphasis on nontechnical courses and less emphasis than formerly on specialized engineering courses, would the effect upon the professional competence of the civil engineer be favorable or unfavorable?"

—13% did not reply. Of those replying 48% believed that the effect would be favorable and 52% that it would be unfavorable.

These are interesting opinions. Many men evidently believe that a civil engineer may actually gain prestige by being less competent "professionally." What is meant by "professional competence" when speaking of a civil engineering graduate? Does the term simply indicate ability to complete an assigned subprofessional task—or the ability and the urge to continue study and development in the civil engineering field? The latter meaning must be implied. A graduate, however, must be able to perform with confidence the first tasks assigned, or he may quickly lose his self-esteem and enthusiasm, and thus not

attain his final goal. First assignments seldom, if ever, utilize the knowledge gained in specialized engineering courses. The study made at the University of Nebraska (in Lincoln) indicates that first assignments for civil engineering graduates usually involve service in surveying and drafting with duties in testing and inspection a very poor third.

If it is concluded that some work should be done in nontechnical courses, many persons are at a loss to know what field of study is desirable. From Table 3(*c*) and Table 4(*c*), it will be noted that all subjects from "psychology" to "art" were marked of "great importance" by at least a few individuals. Of 1,080 replies, 380 believed "psychology" was of "great importance," whereas 28 gave "art" this rating. These are individual expressions of preference, and should be treated as such. It is unfortunate if these individuals were not permitted the privilege of studying in the subject field they believe to be of "great importance."

At the University of Nebraska if an individual student is mentally stimulated by studying courses in philosophy, sociology, literature, or music, he is encouraged to take these courses—with certain limitations, however. A standing committee will continue to study the nontechnical courses offered in the university. A student will be obliged to make his selections from a wide and varied group of subjects approved by that committee and taught at the student's educational level. For example, a student cannot elect a freshman course in philosophy as a nontechnical elective during the junior or senior year, except by special permission from his adviser and the dean. This procedure should oblige the student really to apply himself when he goes outside his field of specialization; and he will be stimulated by his experiences.

There is doubtless another great value in the questionnaire study which has not been developed. Experience at the University of Nebraska leads to the belief that many of the persons returning the questionnaires submitted remarks and written statements in discussion which are helpful and stimulating. It is suggested that these be studied and those of value be reproduced in printed form for distribution.

The use of the questionnaire seems to have been justified, and a great volume of information has been accumulated which will be extremely useful when completely analyzed. The questionnaire has raised additional questions that should be answered. Therefore, it has served a useful purpose.

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DISCUSSIONS

THE PLANNING OF AERIAL PHOTOGRAPHIC PROJECTS

Discussion

BY T. W. NORCROSS, AND JAMES M. CULTICE

T. W. NORCROSS,[3] M. ASCE.[3a]—The U. S. Department of Agriculture (USDA) has been using aerial photographs since approximately 1926. Until the Soil Conservation Service (SCS) started a comparatively large photographic program, little attempt was made to prepare an engineering estimate of the cost of such work. The size of the SCS program, and the personnel available, did not warrant the type of study later made under Mr. Sette's direction, utilizing data collected under the combined programs of all USDA bureaus. The rougher estimates made by the SCS took cognizance of terrain in a manner different from that presented in the paper.

Mr. Sette's approach to this problem is sound and logical. It is based on scale, lens, weather conditions, cost of equipment and materials, cost of operation plus a fair profit, and coverage per photographic exposure. No known tangible item has been omitted. The author concludes correctly that the method needs greater testing and refinement. Certain intangibles not previously included are believed worthy of consideration in refining the method employed in estimating costs.

Table 3(*d*) shows that 634,458 sq miles of photography were used as the basis of computations. This area (or the smaller areas covered in 1939, 1940, and 1941) can be considered as offering an annual photographic program of sufficient size to plan for continuous operation—an attraction to commercial concerns conducive to active competition. A smaller annual program can easily result in the operation of a smaller number of concerns, less competition, and resulting higher unit costs.

Aerial photographs are being sought more and more by a larger number of users as their value is appreciated. Photographic concerns in the United States are no longer dependent almost entirely on the federal government for large contracts and are engaging in photographic operations in foreign countries.

NOTE.—This paper by F. J. Sette was published in March, 1946, *Proceedings*.

[3] Chf., Div. of Eng., U. S. Forest Service, U. S. Dept. of Agriculture, Washington, D. C.

[3a] Received May 31, 1946.

Some of these new customers do not operate on the same basis as the federal government and are willing to pay the price asked to secure the desired material. Competition of this kind can reach such proportions that, excluding the increased costs of operations, either the bid price received will be higher than heretofore; or new concerns must be encouraged to restore competition. Increased competition may result in lower bid prices, but it also raises administrative costs of the contract and delays delivery of satisfactory materials.

In the paper, photography of different types of terrain was taken into consideration only to a limited extent—the effect of power of plane and the higher productivity obtained when flying over sectionized and nonsectionized areas were cited. In the data compiled, the preponderance of areas photographed was agricultural. In this type of terrain, land lines are easily recognizable in sectionized country. Using a heavy plane in nonsectionized country causes difficulties. In photographing mountainous timbered areas, even though in sectionized country, a similar lower productivity would be expected; and, in mountainous country, the heavier plane might easily be necessary to obtain the necessary flying height. When lack of competition is coupled with difficulty of terrain, particularly mountainous terrain, the bidder probably increases the unit cost out of proportion to the difficulty of operation. He knows that few concerns will have the type of equipment required and he will have a better opportunity to secure a price over and above that which would normally result. Necessary flying altitude and amount of available equipment for working at varying flight heights should be considered in the preparation of estimates. In the past many estimates have been made in conformity with the Sette method, with the result that review of the estimate, in conjunction with terrain conditions and required flight height, shows that, because the unit price was considered too low, it has been altered upward to take these difficulties into account.

Photographs are made for a variety of purposes. Generally those secured by USDA are for general land use studies which require the characteristics outlined by Mr. Sette. For mapping purposes photographs of entirely different characteristics may be, and usually are, required. To undertake all kinds of work, the contractors must be equipped to meet the varying requirements. This condition existed at the time the study was made, and the data in the paper were based on the assumption that equipment was adequate. Also, of the volume of work performed at that time, the largest part was for the USDA. Comparative costs to the USDA and mapping agencies would doubtless show a higher unit cost for others because of the smaller volume of work. During World War II, however, many concerns made all their equipment available to the war effort and must re-equip. Many are uncertain of the kind of equipment to purchase. Unit costs will reflect the cost of this equipment, and only through assurances of a large, well-planned program can a high rate of depreciation be avoided.

JAMES M. CULTICE,[4] ESQ.[4a]—Aerial photographic activities were largely suspended for civilian purposes during World War II; and, with the preparation

[4] Chf., Eng. Section, Aerial Photographic Laboratory, Field Service Branch, Production and Marketing Administration, U. S. Dept. of Agriculture, Washington, D. C.

[4a] Received July 10, 1946.

of new bid invitations, the usefulness of such data can again be apparent to contractors and purchasing officials and their engineers. Numerous requests have been received for the clear weather zone map (see Fig. 2).

The validity of the 0.1 cloudiness as an index for available photographic days should be emphasized. The paper indicates 2,973 clear days in contract areas, according to this specification, and 2,877 contract flights within the same period. The difference of 4% is very small and indicates that the probable[5] time necessary to complete an item can be estimated reliably from the standpoint of weather conditions. Although these comparisons are pertinent, it was not intended to convey the impression that flights are made regularly on days of 0.1 cloudiness, and this fact is adequately developed in the paper.

It would be of interest to note the actual award prices for the project areas estimated in Mr. Sette's study. The writer checked the contracts in the southern Indiana section and found a unit price in the successful bid of $2.24 per sq mile on an item of 10,421 sq miles, against an estimated $2.35. In northern Missouri where the weather pattern is more favorable, the contract price of $1.70 for an item of 12,203 sq miles was less than Mr. Sette's lower suggested value of $1.90. These prices were both for 1940 coverage and, thus far, rephotography has not been purchased in these areas. The open, well-developed, and well-sectionized country was of course, conducive to flying economies.

Wartime developments in new aerial photographic planes, cameras, and camera equipment, new flight line control instruments, and better and faster films should make the cost of aerial surveys comparable to the cost of those presented in the paper for areas similar in terrain to the agricultural lands for which the estimating study was originally developed. Existing photo indexes will facilitate the planning of flight lines and should aid the pilot to make better use of flight time through easier recognition of ground detail.

The widespread wartime use of aerial photography and the extension of its usefulness in all fields have greatly increased the demand for aerial coverage for all purposes. The resultant advantages of plane placement and the better utilization of weather conditions in smaller localized areas by concurrent projects will increase efficiency and lower costs. Furthermore, the helpfulness of the cost estimating system is equally effective regardless of price levels. Adequate corrections can be easily applied; and new methods, better equipment, and supplies can be evaluated and their effect on the total final estimate can be computed with some measure of real dependency.

Those concerned with the engineering and administrative phases of the acquisition of aerial photography in the United States Department of Agriculture have frequently noted the value of this study in effecting saving for industry and thereby producing saving for the government and others.

[5] "Days Available for Aerial Photography," by J. M. Cultice, *Photogrammetric Engineering*, Vol. 6, No. 1, p. 32.

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DISCUSSIONS

EFFECT OF STRESS DISTRIBUTION ON YIELD POINTS

Discussion

BY F. P. SHEARWOOD, EDWIN H. GAYLORD, AND L. J. MENSCH

F. P. SHEARWOOD,[3] M. ASCE.[3a]—In commenting on this paper, it should be kept in mind that the true benefit of such a thesis will be realized through the promotion of investigation into the practical and useful application of the findings of the theories. The paper illustrates the fact that "yield" is a most useful and safe quality of steel and an important factor in bringing into play all resistances that oppose deformation (failure). It also indicates that overstrained fibers can be deformed slightly without endangering the structure, provided that other less strained fibers are brought into resistance. Ductility has undoubtedly been the safety valve of commercial designing in cases where poor detailing, eccentric connections, etc., would have caused failure if the elastic factor was the only deformation to be relied upon; but the ductile yield causes redistribution of the stress and so failure is avoided.

In designing or considering continuous spans, and other indeterminate structures as well as the cross bending of beams, modern practice demands that the structures be limited to the specified unit stress in their highest strained path when calculated by the elastic theory. It neglects the fact that, should this path or fiber be strained beyond its elastic limit, the other paths will then exert an increasing proportion of the required extra resistance. This proportion should be about in the ratio of the strain after and before passing the elastic limit—a fairly big ratio.

Investigation of old structures, abused structures, arc welding, badly designed structures, or other weak constructions frequently demonstrates that the yielding properties of steel must have been utilized to prevent failure. This evidence should warrant further investigation leading to the prudent use of this yielding property in the designing of indeterminate steel structures.

The manufacturing of steel and iron is made possible by this property of ductility which is constantly utilized during construction. In fact, much of

NOTE.—This paper by F. G. Eric Peterson was published in April, 1946, *Proceedings*.

[3] Cons. Engr., Dominion Bridge Co., Ltd., Montreal, Que., Canada.

[3a] Received May 6, 1946.

the fabrication of steel would be impossible without this valuable characteristic of yielding. It is inconsistent to assume that yielding of steel is harmless during fabrication or when it relieves the excessive overstressing from faulty detailing and at the same time to prohibit counting on it when designing structures with more than one path of resistance.

It is interesting, and of much consequence, to determine the exact point where yielding will occur; however, the importance of such information lies in practical application to proportioning steel structures.

EDWIN H. GAYLORD,[4] ASSOC. M. ASCE.[4a]—The tests described in this paper are an important contribution to the problem of the behavior to yield in pure flexure and in eccentric tension. It is worth noting that the unit bending stress at which the load deflection curve begins to deviate from the straight line bears the same ratio to the proportional limit in axial tension, as does the unit bending stress at the appearance of Lueders lines to the unit stress in axial tension at the appearance of Lueders lines (Figs. 5 and 10). This phenomenon seems to indicate that the behavior of the extreme fiber in bending, as predicted by the assumption of linear variation of stress, is the same as that of the fiber in axial tension, except for extension of the proportional limit and yield point.

The writer presents an analysis of the data in this paper on the basis of an assumption analogous to that made in the strain-energy theory for combined stresses at a point (21)(22).[4b] For a bar of unit length, the elastic strain-energy equations for axial tension and for pure bending are, respectively:

$$U_a = \frac{1}{2}\frac{P^2}{A\,E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8a)$$

and

$$U_b = \frac{1}{2}\frac{M^2}{E\,I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8b)$$

In Eqs. 8, let $\frac{P}{A} = \sigma_a$, the unit stress in axial tension; and $\frac{M\,c}{I} = \sigma_b$, the extreme fiber stress in bending. Then:

$$U_a = \frac{P^2}{2\,A\,E} = \left(\frac{P}{A}\right)^2 \frac{A}{2\,E} = \frac{\sigma_a^2\,A}{2\,E} \ldots\ldots\ldots\ldots\ldots\ldots (9a)$$

and

$$U_b = \frac{M^2}{2\,E\,I} = \left(\frac{M\,c}{I}\right)^2 \frac{I}{2\,c^2\,E} = \frac{\sigma_b^2\,I}{2\,c^2\,E} \ldots\ldots\ldots\ldots\ldots\ldots (9b)$$

If the unit stress at the yield point in axial tension is σ_{ay}, the yield-point stress in the extreme fiber for a rectangular section in bending is $\sqrt{2}\,\sigma_{ay}$, as computed by the method of W. Kuntze (3), and, using the relation $I/c^2 = b\,h/3$, Eq. 9b becomes:

$$U_b = \frac{1}{3}\frac{\sigma_{ay}^2\,b\,h}{E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

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[4] Prof. of Civ. Eng., The Ohio Univ., Athens, Ohio.

[4a] Received June 3, 1946.

[4b] Numerals in parentheses, thus: (21), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

For a rectangular bar of the same cross section at the yield point in axial tension, Eq. 9*a* gives:

$$U_a = \frac{1}{2}\frac{\sigma^2_{ay}\, b\, h}{E} \qquad (10b)$$

If the shape of the cross section is a symmetrical diamond, the Kuntze method indicates an extreme fiber stress of precisely $2\,\sigma_{ay}$ at the yield point in bending. If b and h are taken as the width and depth, respectively, $A = b\,h/2$ and $I/c^2 = \frac{b\,h^3/48}{h^2/4} = b\,h/12$; and, from Eqs. 9,

$$U_b = \frac{1}{6}\frac{\sigma^2_{ay}\, b\, h}{E} \qquad (11a)$$

and

$$U_a = \frac{1}{4}\frac{\sigma^2_{ay}\, b\, h}{E} \qquad (11b)$$

Based on Eqs. 10 and 11, the assumption can be made that, at the yield point in pure bending, the strain energy in a given length of bar is equal to two thirds of the strain energy of a bar of the same length and cross section at the yield point in axial tension. The yield point is defined as that point at which Lueders lines appear. Then, from Eqs. 9, if σ_{by} is the extreme fiber unit stress in bending at the yield point:

$$\frac{1}{2}\frac{\sigma^2_{by}\, I}{c^2\, E} = \frac{1}{3}\frac{\sigma^2_{ay}\, A}{E} \qquad (12a)$$

and, since $I = A\, r^2$,

$$\sigma_{by} = \sqrt{\frac{2}{3}}\frac{c}{r}\sigma_{ay} = 0.816\frac{c}{r}\sigma_{ay} \qquad (12b)$$

It is obvious that Eq. 12*b* will check the Kuntze method for the rectangular and diamond cross sections precisely. Table 5 gives a comparison of the

TABLE 5.—COMPARISON OF EQ. 12*b* WITH THE KUNTZE METHOD

Section	c	r	σ_{by}/σ_{ay}		Col. 4 / Col. 5
			Eq. 12*b*	Kuntze	
(1)	(2)	(3)	(4)	(5)	(6)
Circular	R[a]	$R/2$	1.632	1.644	0.993
Standard 6-in. pipe	3.3125	2.25	1.202	1.207	0.996
36WF230	17.94	14.88	0.984	1.047	0.940
36WF150	17.92	14.29	1.024	1.039	0.986
18WF47	8.95	7.30	1.001	1.041	0.961
8WF17	4.00	3.36	0.972	1.050	0.926

(a) R equals radius.

results of Mr. Kuntze's method and the results of Eq. 12*b* for a circular section, a tube, and a few wide-flange sections. For some of the wide-flange sections, the bending yield stress, as calculated by Eq. 12*b*, is indicated as being slightly less than the yield-point stress in axial tension. It is undoubtedly true that

the two-thirds factor in Eq. 12*b* is not constant, but dependent on the shape, and that it approaches unity as r approaches c. Only in this way could the equation check the hypothetical beam in which all the material is concentrated along two lines parallel to, and equidistant from, the neutral axis; and such a section would surely be expected to yield at the yield stress for axial tension. Nevertheless, Cols. 4 and 5, Table 5, agree very well. In fact, by changing the factor 0.816 to 0.85, Eq. 12*b* will predict the bending yield, for the sections which the writer has checked, within $\pm$ 4% of the values obtained by the Kuntze method.

Eq. 12*b* provides an extremely simple method of calculating the yield stress in bending and, in so far as the two have been compared, gives substantially the same results as does the Kuntze method. Should subsequent tests establish the increase in bending yield as being reliable enough to warrant a change in design procedure, and also establish the strain-energy hypothesis, choice of a section to resist bending can be made without using Eq. 12*b* directly. If the yield stress as given by Eq. 12*b* is equated to the stress calculated from $\sigma = \frac{M\,c}{I}$,

$$\sqrt{\frac{2}{3}}\frac{I}{r} = \frac{M}{\sigma_{ay}} \qquad (13)$$

In Eq. 13, the quantity $\sqrt{\frac{2}{3}}\frac{I}{r}$ is the section modulus. If a specification working stress is used in Eq. 13 instead of σ_{ay}, a uniform factor of safety with respect to yield would result for all cross sections with two axes of symmetry, regardless of shape. An extensive series of tests on both small and large sections would seem profitable, to try to establish firmly the apparent increase in the bending yield-point stress. Tests on sections which are unsymmetrical about the principal axis normal to the plane of bending would also be valuable, to determine the effect of this factor on the increase in bending-yield stress.

In extending the strain-energy equation to the case of eccentric tension, at least two assumptions are possible. One is expressed by the equation,

$$\frac{\sigma^2_b\, I}{c^2\, E} = k^2_1 \left(\frac{\sigma^2_{ay}\, A}{E} - \frac{\sigma^2_a\, A}{E} \right) \qquad (14a)$$

The factor k^2_1 may be variable, but it must have the value 2/3 for $\sigma_a = 0$, to agree with Eq. 12*a*. Another possible assumption is expressed by

$$\frac{\sigma^2_b\, I}{c^2\, E} - \frac{\sigma^2_a\, A}{E} = \frac{k^2_2\, \sigma^2_{ay}\, A}{E} \qquad (14b)$$

In Eq. 14*b*, k^2_2 is definitely variable, since it must have the value 2/3 for $\sigma_a = 0$, and the value unity for $\sigma_b = 0$.

To obtain an expression for the maximum yield stress, the relation,

$$\frac{\sigma_b}{\sigma_a} = \frac{M\,c/I}{P/A} = \frac{e\,c}{r^2} \qquad (15a)$$

can be used to eliminate σ_b from Eq. 14*a*; thus:

$$\left(\frac{e}{r}\right)^2 \sigma^2_a + k^2_1 \sigma^2_a = k^2_1 \sigma^2_{ay} \quad \ldots\ldots (15b)$$

from which

$$\sigma_a = \frac{k_1 \sigma_{ay}}{\sqrt{k^2_1 + (e/r)^2}} \quad \ldots\ldots (16a)$$

Similarly, from Eq. 14*b*,

$$\sigma_a = \frac{k_2 \sigma_{ay}}{\sqrt{1 + (e/r)^2}} \quad \ldots\ldots (16b)$$

The maximum stress σ_{my} is given by

$$\sigma_{my} = \sigma_a + \sigma_b = \sigma_a (1 + e\, c/r^2) \quad \ldots\ldots (17)$$

and, therefore, from Eqs. 16:

$$\sigma_{my} = k_1 \frac{1 + e\, c/r^2}{\sqrt{k^2_1 + (e/r)^2}} \sigma_{ay} = \frac{1 + e\, c/r^2}{\sqrt{1 + (1/k^2_1)(e/r)^2}} \sigma_{ay} \quad \ldots\ldots (18a)$$

and

$$\sigma_{my} = k_2 \frac{1 + e\, c/r^2}{\sqrt{1 + (e/r)^2}} \sigma_{ay} \quad \ldots\ldots (18b)$$

For the specimens in Table 2, $e = 0.3$ in.; c/r^2 ($= 6/h$ for a rectangular section) $= 6/1.800 = 10/3$; and $r^2 = h^2/12 = 0.27$. Assuming an increase in yield stress of about 41%, or, say, $\sigma_{my} = \sqrt{2}\, \sigma_{ay}$, as is indicated for specimens III and IV of Table 3, and, substituting these values in Eqs. 16, $k^2_1 = 1/3$ and $k^2_2 = 2/3$.

Since k^2_1 has the values 2/3 for pure bending and 1/3 for the eccentric tension specimens, it seems reasonable to assume that it will approach zero with e. This variation can be expressed very simply by the equation,

$$k^2_1 = \frac{2}{3} \frac{a\, x}{2\, x + 1} \quad \ldots\ldots (19a)$$

in which the constant a depends on the choice of the independent variable x. The variation of k^2_1 might be assumed to be with e, or with the dimensionless quantities e/r, e/h, $e\, c/r^2$, etc. It seems appropriate, however, to assume the variation to be with $e\, c/r^2$, since this quantity is the ratio of σ_b to σ_a. With this assumption, Eq. 19*a* reduces to

$$k^2_1 = \frac{2}{3} \frac{e\, c/r^2}{e\, c/r^2 + 1} \quad \ldots\ldots (19b)$$

in which $a = 1$ was calculated on the assumption that $k^2_1 = 1/3$ when $e\, c/r^2 = 1$ (the values of these variables for the specimens of Table 3). The simplicity of Eq. 19*b* is encouraging, and the relation seems even more plausible when $e\, c/r^2$ is replaced by its equivalent σ_b/σ_a. This substitution leads to

$$k^2_1 = \frac{2}{3} \frac{\sigma_b}{\sigma_a + \sigma_b} \quad \ldots\ldots (19c)$$

Since the value of k^2_2 is the same for the specimens of Table 3 as it is for pure bending, and yet must approach unity as e approaches zero, it is more difficult to assume a variation of k^2_2 without further data.

In seeking a possible explanation for the larger increase in yield stress for specimens I and II of Table 3, it might be assumed that there was an accidental eccentricity across the thickness of the specimen, since a constant eccentricity would have a greater effect on the thinner specimens. Eq. 18*a* is easily extended for eccentricities in two directions, and results in:

$$\sigma_{my} = \frac{1 + (e\,c/r^2)_1 + (e\,c/r^2)_2}{\sqrt{1 + (1/k^2_1)[(e/r)^2_1 + (e/r)^2_2]}}\,\sigma_{ay} \quad \ldots\ldots\ldots\ldots (20)$$

The value of k^2_1 can be assumed to be given by the equation

$$k^2_1 = \frac{2}{3}\,\frac{(e\,c/r^2)_1 + (e\,c/r^2)_2}{(e\,c/r^2)_1 + (e\,c/r^2)_2 + 1} \quad \ldots\ldots\ldots\ldots (21a)$$

or

$$k^2_1 = \frac{2}{3}\,\frac{\sigma_{b_1} + \sigma_{b_2}}{\sigma_a + \sigma_{b_1} + \sigma_{b_2}} \quad \ldots\ldots\ldots\ldots (21b)$$

In Eqs. 20 and 21, the subscripts denote the principal axes of the bar.

Assuming that there is an accidental eccentricity of 0.001 in. in the direction of the thickness for each specimen of Table 3, Eqs. 20 and 21*a* yield: For specimen I—

$$\sigma_{my} = 1.54\,\sigma_{ay} \quad \ldots\ldots\ldots\ldots (22a)$$

for specimen II—

$$\sigma_{my} = 1.47\,\sigma_{ay} \quad \ldots\ldots\ldots\ldots (22b)$$

for specimen III—

$$\sigma_{my} = 1.44\,\sigma_{ay} \quad \ldots\ldots\ldots\ldots (22c)$$

and, for specimen IV—

$$\sigma_{my} = 1.43\,\sigma_{ay} \quad \ldots\ldots\ldots\ldots (22d)$$

The agreement with the increases shown in Table 3, Col. 6, is surprisingly good. Perhaps Mr. Peterson will state in his closing discussion whether such an accidental eccentricity might have existed.

It will be of interest to investigate Eq. 18*a* for maximum or minimum values. Differentiating the equation with respect to e, and remembering that k^2_1 is also a function of e, with $\frac{d(k^2_1)}{de}$ to be obtained from Eq. 19*b*; a maximum or a minimum for the value of σ_{my} is found to occur for the value of e given by

$$e = \frac{4\,c}{3} - \frac{r^2}{c} \quad \ldots\ldots\ldots\ldots (23)$$

For a rectangular section, Eq. 23 gives $e = h/2$. Then $k^2_1 = 1/2$ and $\sigma_{my} = (4/\sqrt{7})\,\sigma_{ay} = 1.51\,\sigma_{ay}$. This is evidently a maximum. The result is perhaps disappointing, as one might expect the increase in yield to be greatest for pure bending. However, considering that Mr. Peterson's tests show the same increase for eccentric tension with $e = h/6$ as for pure bending ($e \rightarrow \infty$) and that there can be no increase for $e = 0$, the existence of a maximum value

seems plausible. For a circular section, Eq. 23 gives $e = 13\,R/12$. Then $k^2_1 = 13/24$ and $\sigma_{my} = 1.71\,\sigma_{ay}$. For an 18WF47 the maximum value occurs for $e = 5.98$ in. and is $1.15\,\sigma_{ay}$.

The increase in yield stress for eccentric tension could be determined by another method. It is common practice in aeronautical structural engineering to use so-called stress-ratio curves for combined stresses (23)(24). The specification of the American Institute of Steel Construction for combined axial and bending stress (25), expressed by the inequality $f_a/F_a + f_b/F_b < 1$, is an example. Although there are insufficient data from which to determine stress-ratio equations for the problem in Mr. Peterson's paper, it is interesting to observe that, for a rectangular section, the equation:

$$\frac{\sigma_b}{\sigma_{by}} = 1 - \left(\frac{\sigma_a}{\sigma_{ay}}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

checks specimens III and IV of Table 3. For specimen IV, $\frac{\sigma_a}{\sigma_{ay}} = \frac{31{,}750}{45{,}000} = 0.706$. Substituting this value in Eq. 24, $\frac{\sigma_b}{\sigma_{by}} = 0.498$. The actual value of $\frac{\sigma_b}{\sigma_{by}}$ from Table 3 is $\frac{31{,}750}{45{,}000\sqrt{2}} = 0.499$.

The equations developed in this discussion are admittedly speculative, and are based on incomplete data. However, granting the possibility of the accidental eccentricity of 0.001 in. previously discussed, the results seem sufficiently good to justify further verification and development.

L. J. MENSCH,[5] M. ASCE.[5a]—The related phenomena of plasticity, ductility, time yield, elastic failure, and fatigue are of great interest to manufacturers of products made from high and low yield-point steel, aluminum and its alloys, copper, gold and silver and other metals, plastics, and especially those of wood, cast iron, and reinforced concrete. The aforementioned phenomena lead to serious deviations from the behavior of materials as visualized by Hooke's law which is the foundation of the modern theory of elasticity.

The problems created by these deviations have been the subject of extensive and costly research during the last two hundred years by large users of the various materials. The first important series of tests known to the writer was made by the French government around 1750 on timber, followed about 50 years later by more thorough tests by the British Admiralty (26).

In the latter tests, Thomas Barlow subjected a great number of full-sized beams as used in ships to uniform loads (with cannon balls) and noted deflections and ultimate loads, when the beams were simply supported. Then he fixed similar sized beams by cast-iron shoes in heavy masonry walls and loaded them to failure. According to the theory of elasticity, the fixed beams should have carried a load 50% greater than the simply-supported beams; actually, without question or doubt, they carried 100% more. In modern terminology

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[5a] Received June 27, 1946.

it would be stated that failure occurred under a bending moment of $w\,L^2/16$. Beams fixed at one end only failed under a bending moment of $w\,L^2/10.8$. Mr. Barlow also tested beams of triangular sections, both with the edge up and the edge down, simply supported and fixed, and found very serious deviations from the results as taught by the theory of elasticity. He discovered similar discrepancies when testing beams of metals and glass, and further referred to tests by Thomas Young (Young's modulus of elasticity) on cast iron, where even greater deviations occurred. Mr. Barlow boldly asserted that the theory of elasticity assumed properties of materials not to be found in nature and led to underestimating the real strength of beams.

Teachers of the theory of elasticity did not pay much attention to Mr. Barlow and the most important series of tests which were ever made up to that time, and one famous work on the theory of elasticity (27) refers to the speculations of Mr. Barlow as the "nadir" of English thought on applied mechanics.

In the nineteenth century the fact that the theory of elasticity underestimated the strength of materials in most cases was the subject of reports by the laboratories of Hodgekinson in Manchester, England; by Sir Thomas Fairbairn also of Manchester; by David Kirkaldy and Son of London, England; by Professor Tresca of Paris, France, and by his pupil A. Considere; and by A. Kick of Prague, Czechoslovakia. No attention was paid by the teachers of the theory of elasticity to all these reports, since the teachers were too much intrigued by the elegance of their methods and too much encouraged by the success of their pupils in the construction of bridges and halls of unprecedented spans. It is, of course, true that the theory of elasticity leads to safe results; but, as Mr. Barlow and others showed, more economical designs may be obtained by considering the deviations of the materials from behavior like that specified by Hooke's law under high stresses.

This phase of the strength of materials has been treated in papers and discussions before the Society in the last 75 years. Although practical engineers greatly benefited by these papers, teachers did not. In December, 1914, the writer published a working theory of the strength of reinforced concrete beams based on conditions near the ultimate load (28)(29). A similar theory for steel structures and other materials was published in 1918 (30). Not much attention was paid to these and other papers by the writer on this subject; but, when the German engineers in the early 1930's found that no work could go ahead in that impoverished land unless fewer materials were used than formerly, a furious campaign was started in German literature because of a supposed discovery that a theory based on conditions of ultimate load would justify higher stresses on materials of construction.

The author of this paper derived his inspiration from these overseas sources and imitated their crude tests (afflicted with an error of about 20% due to friction, as can be noted by comparison with the tests made by the Special Committee of the Society on Column Research (31), at Madison, Wis.).

Nothing new is offered in this paper and it may be of value only to call the attention of teachers to the insufficiency of the theory of elasticity to describe the strength of materials accurately.

Bibliography.—

(3) **"Ermittlung des Einflusses ungleichförmiger Spannungen und Querschnitte auf die Streckgrenze,"** by W. Kuntze, *Der Stahlbau*, Vol. 7, 1933, Heft 7, p. 49.

(21) **"Strength of Materials,"** by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., Pt. II, 1930, p. 708.

(22) **"Failure Theories of Materials Subjected to Combined Stresses,"** by Joseph Marin, *Transactions*, ASCE, Vol. 101, 1936, p. 1170.

(23) **"Strength of Aircraft Elements,"** by Army-Navy-Civil Committee on Aircraft Design Criteria, 22 October, 1943, p. 1–18.

(24) **"Theory of Elastic Stability,"** by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936, p. 489.

(25) **"Steel Construction,"** A.I.S.C., 4th Ed., 1942, p. 275.

(26) **"The Strength of Timber,"** by Thomas Barlow, Woolwich Arsenal, London, England, 1814.

(27) **"A History of the Theory of Elasticity,"** by I. Todhunter and K. Pearson, Univ. Press, Cambridge, England, 1886.

(28) *Journal*, A.C.I., December, 1914, p. 28.

(29) *Proceedings*, A.C.I., Vol. 33, 1937, p. 498–1.

(30) *Transactions*, ASCE, Vol. LXXXIII, 1919–1920, p. 1667.

(31) **"Steel Column Research,"** Final Report of the Special Committee, *Transactions*, ASCE, Vol. 98, 1933, p. 1376.

Correction for *Transactions:* In Eq. 6 of the paper, reverse the ratio $\frac{\rho_1}{\rho_2}$.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

SOME THOUGHTS ON ENGINEERING EDUCATION

Discussion

BY E. S. BOALICH, RUSSELL C. BRINKER, I. OESTERBLOM, SAMUEL T. CARPENTER, LYNN PERRY, AND L. E. GRINTER

E. S. BOALICH,[3] ESQ.[3a]—Something has been radically wrong with engineering education as two groups of engineers will testify. In the language of the paper (see heading, "The Practice of Engineering") these groups are:

Group I—The wizards in matter, who "fell down" when they came to men, money, or management. Mr. Baker's recommendations should take care of most members of this group perfectly.

Group II—Those who were never too proficient in M-1, including some who might have been wizards in M-2, M-3, or M-4 (or in art or literature!).

An added suggestion is to go back just one step behind Mr. Baker's starting point and to do as the steel plant does when it analyzes carefully each batch of iron ore before processing it.

By intelligent screening in the first college year, of a five-year course, "ore" which would never produce "good steel," but which might be in demand for "ballast," could be properly diverted; and as a dividend an occasional "gold nugget" would turn up which might otherwise have been forever lost in "the fiery furnace."

RUSSELL C. BRINKER,[4] ASSOC. M. ASCE.[4a]—To one who has returned to the teaching profession after some years in the armed forces where the results of different types of training could be observed, this paper presents many points of interest. Consideration of the most desirable content of engineering curricula is occupying the time of many deans and heads of engineering departments. Various engineering and educational societies have gone on record as favoring

NOTE.—This paper by Donald M. Baker was published in April, 1946, *Proceedings*.

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[3a] Received May 10, 1946.

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[4a] Received June 10, 1946.

an increase in the time spent on humanities courses. This increase means either elimination of some technical courses, or expansion to a five-year program. The University of Minnesota at Minneapolis and Ohio State University at Columbus are representative of schools adopting the latter plan. Doubtless many others will follow, although a desire to permit returned veterans to graduate in as short a time as possible will retard this trend temporarily. In past years, fear of loss of students who could not afford the additional year delayed the change-over. Problems arising in connection with a change to the five-year program illustrate some of the difficulties to be overcome if Mr. Baker's recommendations are to be accepted.

Reduction in time, or elimination of some courses seeking primarily to impart skills, is suggested. Many graduates use these "skills" to begin or to continue their work. In normal times an estimated 60% of the civil engineering graduates of the University of Minnesota start their engineering careers in surveying or drafting. Without such skills, these men would be rodmen instead of instrumentmen; tracers instead of draftsmen. The self-confidence and additional compensation resulting from extended work in surveying and drafting might well keep the young graduate in his profession. It is not proposed that all these men remain in "skilled" jobs. Nevertheless, if advanced theory and specialty courses are to be deferred until some time after graduation then the engineering graduate should be well trained—if that properly describes his status—for the job that he will have a 60% chance of taking immediately after graduation.

Drawing and surveying courses fill a definite need for some work in engineering during the first two years of college if student interest is to be maintained. This point becomes increasingly important if curricula are extended over five years and the professional courses held back a year. Pre-engineering requirements, like prelaw or premedical standards, might be desirable. It is undoubtedly true, though, that many men who apply themselves well in the later professional courses are concerned only with obtaining marks high enough in the preliminary work to insure entry to the professional schools.

Substitution of two-year programs of the trade school variety is not generally favored by the faculty or acceptable to a large percentage of students. The unlimited educational opportunities so characteristic of the United States inspire engineering students to try for the status of a four-year graduate or nothing.

It has been stated frequently that many engineers are unable to express themselves well, or to write good reports. One reason for these handicaps is probably the failure of most engineering faculty members to insist upon the continuation in all later subjects, of the standards set up in freshmen English courses. Professors and instructors are apparently not willing to take time to correct spelling and sentence construction and stress the need for improvement. Students, of course, feel that freshmen English courses are hurdles to get over or around, and that they are of no practical use in succeeding work, where "anything goes." Too often anything does go!

There is a tendency for faculty members to be specialists, perhaps engrossed in some isolated research problem that has little or no connection with class

work. Teaching therefore becomes boring, and all interest is lost in the meticulous checking of problems and reports by students. National reputations for research or publications are not maintained by long hours of student consultations or paper grading. On the other hand, few professors in mechanical drawing, surveying, mathematics, and freshmen English courses (which require "plugging" at details) attain any great prominence; yet to be an inspiring teacher in these subjects—as compared with structural design courses, for example—really "takes something"!

Lack of the "dollar sense" is a highly valid criticism of new graduates in engineering—and of many experienced ones too. How many instructors or full professors touch even lightly on the mill price per pound of a steel beam or column that has been designed in class, and follow with statements on fabrication and erection costs? How many mention the cost of formwork and discuss its influence in selecting an economical design? Less than one hour's time spread over a semester would develop the ever-present student interest in "How much does it cost?" Probably not one surveying instructor in ten has discussed the cost of operating a survey party, including interest and depreciation on equipment, etc. Lowered professional standards are certain to result as shown by a recent case when a well-qualified senior student made a detailed land survey for $5—his hourly wage probably being less than 25¢ per hr. Such practices may well be extended into succeeding work unless checked early by proper instruction.

Too few students know about the advantages and requirements for registration as a professional engineer. Medical students are indoctrinated early in the responsibilities imposed by the degree of Doctor of Medicine. Engineering students should begin hearing in the freshman year, and frequently thereafter, about professional registration so that it will become a goal secondary only to graduation. Unfortunately some faculty members are not registered and may hesitate to assume a "Do as I say, not as I do" attitude.

Engineering faculties should emphasize in junior and senior classes the need for continuing study after graduation. This may be done by mentioning in each course some of the important theories and problems omitted because of lack of time.

Except in large cities or college towns, postgraduate work must be conducted by correspondence courses since the absence of faculty and sufficiently large groups interested in a particular subject make the cost prohibitive. A "Center for Continuation Study" established at the University of Minnesota typifies the facilities needed in many cities to provide wide coverage. Correspondence study is commendable and the material covered "sticks" with the individual; but it does not have the advantage of personal contact with a teacher, and it is likely to be long drawn out and time consuming. Human nature being what it is, relatively few men will progress steadily unless a rigid schedule is set up. Class hours provide this schedule. Personal observation of the attitude of many men in the armed forces indicated a desire to take correspondence and classroom work but also a frequent lack of "push" to complete it.

Most important, a tangible goal must be established that a graduate can hope to reach within a reasonable and specific time. Several three-year or

four-year "plans" could be set up. At the end of the first, a degree of Master of Civil Engineering might be the goal. Following a second plan the professional degree of "Civil Engineer" might be awarded. At the present time only two schools in the United States confer the "professional" degree (not in civil engineering) on a graduate at the end of four years. In other institutions, however, the requirements form a confused picture.[5] Some schools give it as an honorary degree, some for graduate work and a thesis, and some for a thesis only. Usually a minimum period of from four or five years after graduation is specified. On the basis of a broad outline to permit numerous options and substitutions, a definite program could be planned which would permit a graduate to study by correspondence and in the classroom, so that he would know how and when he might obtain a degree. Isolated courses are helpful and desirable but they are far from being a solution to the problem of continuation of study.

This program might be extended still further. The author mentions the progression through the three M's—matter, men, and money. At about the time that a man has finished work for the professional degree he should be interested in the second and third "M's." A third four-year plan could be laid out leading to a degree in business or administration. Education is not a matter of regimentation in thought. It is, however, a problem in regimentation of scholastic resources and an individual's time which planned programs will help to solve.

Personal experience with men taking isolated evening courses indicates that a troubled conscience rather than a desire for knowledge impels some of them to register. A man feels he is not advancing, so he registers for a course, pays his fees, and sits back with the inner voice silenced. He may come to class to be amused or to acquire bits of information the easy way without studying. Little or no work is done outside class and absences are frequent. This type of individual will always increase evening enrolment in no-credit "occasional" courses, and perhaps he benefits enough to justify the tuition fee. A definite planned program, however, as opposed to isolated courses or lectures based on sales appeal, should reduce or eliminate the number of such men.

Engineering theory and computations are a difficult "grind" after a day spent on similar problems in field or office. It is a personal belief that, once past the fundamental humanities courses, a graduate would find it easier to begin and continue a program of study, either by night classes or by correspondence, in that field rather than in engineering. Schools contemplating preparation of long-range programs for graduates might consider this point when "pruning" engineering courses and "grafting" humanities subjects. Physicists have moved into many fields formerly claimed by engineers. This trend will continue if basic sciences and technical work are sacrificed to the humanities.

Many developments of the extensive service training programs should be introduced in high school and college teaching. The "Instructors Guide"[6] gives some hints that professors may long since have forgotten. A theory of service teaching is that students remember only 10% of what is heard in lec-

[5] "Status of Engineering Degrees," by O. M. Leland, *Proceedings*, S.P.E.E., Vol. XLVI, 1938, p. 322

[6] "Instructors Guide," Marine Corps Schools, Marine Barracks, Quantico, Va.

tures. This is a point that "long-winded" faculty members might well take to heart. Emphasis on slides to reduce time and improve instruction is needed. Exposure to Japanese language instruction by the newer method of conversation first and grammar later was convincing proof of its advantages.

Faculty members probably know the needs of industry for new graduates but they do not know much about the market for older engineers. Graduating students are likely to consult with professors about that all-important first job but seldom request advice later when a change of position is contemplated. This may result from being too far from the campus, or from a desire to make one's own decisions. Possibly more recent contacts in the particular field are considered to carry greater weight. Also, a representative cross section of graduates does not return for consultations with the faculty. One of the compensations of teaching is having former students drop in for social calls or advice. Unfortunately, except for a few job hunters, only those in the higher scholastic brackets do so, and the problems of men who just "squeaked" through are not brought to light. During depression years, the school employment offices, run at least informally by most engineering schools, have many calls. Mainly, however, service to the new graduate is their primary function.

Mr. Baker's data show that many engineers leave their profession—more so than in the case of doctors, dentists, and lawyers. This is natural since an engineer is not handicapped by such a change. The training he has received fits him for many types of work. Doctors, dentists, and lawyers have a heavier investment in time and money for their highly specialized training, hence, there is a greater incentive to remain in their profession. Many college graduates in education, business, etc., also desert their spheres of specialized training for the "greener grass" of other fields. Men trained in engineering find it desirable to seek other employment because of the low salaries paid to engineers. This is a serious condition facing the profession. Fortunately, graduates of engineering schools prove capable of competing out of their own fields with personnel from business and arts colleges. This seems to indicate the type of training given in engineering schools, even if it develops only the habit of hard work and long hours, is reasonably satisfactory.

Mr. Baker's paper has undoubtedly crystallized the thoughts of many engineers and educators. The collection of supporting data is a worthy contribution.

I. OESTERBLOM,[7] M. ASCE.[7a]—"Thoughts on Education" is a very wide field, and it is pleasant that the author has strolled deliberately beyond its normal limits. Since the engineer has just proved to the world by his extraordinary performance in war that he is a giant in both mind and body, the education that lies beneath all his achievement is interesting. It must be good, and yet it might be better.

It must be good—for otherwise the performance would not have been so stupendous—and it might be better—for that is the spirit of the engineer.

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[7a] Received June 17, 1946.

There is always a better way to do things. Were this not so there would be an end to progress.

A review by anybody with a keen and open mind must therefore be instructive, constructive, and of good purpose, and this review by Mr. Baker is especially so. The paper meets all these specifications and it also looks well into the future. However, something seems to be missing. There is a forgotten field which needs to be explored.

When the Hoogley River Bridge was built a few years ago from Calcutta to Howrah in India, it came into being through intense economic pressure which had accumulated over a number of years. As far as the bridge was concerned those years had not been idle ones. There was a temporary pontoon construction across the river which had been forced to serve for a very long time. Over the bridge had gone for years very heavy traffic between two large cities; and through the bridge there was also heavy river traffic. Each was increasing every year—dangerously so. Thus, the social and economic statesmen—and the engineers—had been set to work to plan a new bridge. These groups worked hard for many years because the problem was a difficult one. A huge structure was required, and a flexible one which would take care of the two antagonistic streams of traffic. Much space on land was needed, and all the land was occupied. The river was treacherous and "emotional," and it ran over a soil that was soft and shifting.

Thus, some official authorities and many consultants produced a number of bridge plans, all of which were excellent and beautiful and one of which was finally selected under the economic pressure of the conditions.

This case is selected as an illustration because it is so broadly typical and because substantially the same conditions exist for all the large engineering projects as they also do in a small way for the minor tasks. The economist—especially the contentedly Marxian—has a fine example of the economic pressure that governs the destinies of man—and he can see that only. However, there is something even more fundamental at work.

Thus one arrives at the forgotten field—to the creative urge in man; to the dreams of constructive work to be done; to the dreams of the engineer, which are born in his mind and which he sets down on paper in his waking hours. They are brilliant flashes, sometimes, but more often they came into being slowly and painfully, borne from experience and knowledge and from that curious urge in all men to create new things out of scattered stones.

The point that this commentator wishes to stress is that these dreams are the most important fact in engineering. Also that in both education and discussion this is officially a forgotten field. In fact, it is even worse than forgotten.

In any school or in any office "thou shalt" or "thou shalt not" predominates. The professor does his utmost to train a flexible mind into a rigid form; and, after school is finished, whatever is left of flexibility is hardened by office standards and specifications. It is not to be inferred that all these tokens of rigidity should not be and that they are not useful, but they should be accepted as servants of the mind and not as prisons to hold the mind captive or as urns in which the mind may be slowly petrified.

How is it then that, even under these conditions, so much is being created and that the performance in the war was so unusually and successfully creative?

This is a most interesting question. For in the welter of minds there are always a few that refuse to be held captive; or, if so held, that are ready to explode into action the moment they sense a call of urgency. They are just like the "Prodigal Judge" of Old Tennessee—besotted and contemptible as long as the soil seemed like a prison but magnificent and a supreme leader when there was work to do.

There is tragedy in the other side of the picture—and that is the prime justification for these comments. Fine minds with a fine education and high diplomas, frozen forever to the drafting boards—once minds ready for constructive dreams but condemned to the drudgery of "routineering." They are now mere slaves of formulas which the mind of a clerk would be happy to use and set down on paper and which he could do even better.

The world therefore is so much the poorer.

These minds have been destroyed by the authoritative attitudes of professors and chiefs, because so many of these have been fascinated by the exterior forms and have not yet seen the rich field of thoughts and dreams beyond. This, therefore, is an invitation to reorganize both material and methods accordingly. More imagination is needed in both school and office as is a better understanding that convention and rigid form should be aids and not masters and that each should be destroyed when it can no longer serve as an element of efficiency and progress.

Should there be special courses in creative imagination in the universities? Perhaps there should be in the future but not now—the time is not ripe. If such courses were started now, some professors would drop dead from fright at such sacrilegious interference with the worshipped forms and formulas. That result would be even more disastrous, for good conventions set the limits to the dreams and without them there would be, at best, beautiful chaos. By preference the thought of the constructive dream should always be present in everything, including seemingly dry calculus, that the teacher tries to convey to the student's mind and in all things the chief of an office attempts to create.

How much more attractive would be the work of each man if such inspiration were always present to pull the potential leader out of his weary armchair of humdrum formalism.

SAMUEL T. CARPENTER,[8] ASSOC. M. ASCE.[8a]—The clear thinking evidenced in this paper relative to the important problems faced by the engineering schools and engineering societies should be commended. It is to be hoped that in the future there will be more articles by men active within the profession, as their ideas are valuable in formulating the objectives of engineering curricula and standards for professional training.

The factual information in the paper represents important data and furthermore shows that a fundamental, rather than a highly specialized, training in engineering is desirable. It is interesting to note that the questionnaire study performed jointly by the Society's Committee on Engineering Education and

[8] Chairman, Civ. Eng. Dept., Swarthmore College, Swarthmore, Pa.
[8a] Received June 28, 1946.

the Cooperative Committee on Civil Engineering Education of the Society for the Promotion of Engineering Education verifies Mr. Baker's contentions. The survey indicates that, in general, engineering graduates are very weak in their ability to address a private group or public gathering, and very poor in writing a letter or report. They were also found to be practically deficient in their interest in public affairs. These deficiencies and other deficiencies noted by this survey can be largely corrected by a general improvement in engineering teaching staffs.

Mr. Baker appears to emphasize the part that engineering schools should take in the development of men after graduation. The writer heartily agrees with this, and believes that most engineering schools do attempt to do all they can for their graduates. However, the writer believes that the Society, as well as all other engineering societies, must take an increased interest and be a key factor in the professional development and postgraduate education of the young graduate. There are so many ways in which the engineering societies can and should assist in the professional development of the graduate that each and every one should have a full-time staff man to direct all its relations with engineering education and professional training. The Society should make a continuous survey of the needs of industry for graduates with a civil engineering education.

The Society should also sponsor a campaign directed toward employers of civil engineering graduates to encourage the treatment of the graduate as a prospective professional engineer. The Society should not approach engineering employers and ask them to be altruistic, but instead should sell them on the merits of the professionally trained man in such a way that employers will give the graduates an opportunity to rise from the subprofessional status in which many find themselves. Employers could be urged to adopt a training program which would indoctrinate the young man in all phases of work encountered in the engineering office. This is essential since, with the advent of the "Engineer-in-Training" status, the young man must be prepared to take his professional examination within a period of from five to ten years.

The Society has recommended that a more fundamental education be given by civil engineering schools. It is believed that this places a responsibility upon the Society to use the Society organization more effectively as an instrument for professional development. Such a result could be brought about by a coordinated and unified effort on the part of the Society's Committees on Engineering Education, Juniors, and on Student Chapters; the civil engineering members of the Committee on Professional Training of the Engineers' Council for Professional Development; and other agencies of the Society. The over-all problem is so immense that space does not suffice to discuss it here. Accelerated trends in science and society call for education throughout a man's career. It is hoped that Mr. Baker's excellent article will stimulate the engineering societies, as well as the engineering schools, to action.

LYNN PERRY,[9] M. ASCE.[9a]—By the publication of this paper, the author has done much to help engineers see themselves as others see them. He has

[9] Designing Engr., Dept. of Water and Sewers, Miami, Fla.
[9a] Received June 29, 1946

pointed out the usual personal shortcomings of many of the members of the profession. Then with grace and tact he has placed the blame squarely on the shoulders of the professors of engineering who are responsible for the curricula. Curricula considerations are very much more involved and far reaching than the engineer practicing his profession realizes. Perhaps some professors of engineering will follow the suggestion of T. R. Agg,[10] M. ASCE, and "talk back." Probably no more appropriate time could have been chosen to reopen this subject.

In the opening paragraphs, the author, after admitting technical competence and skills, frankly states that the colleges have been training rather than educating the student body. He is concerned about the pattern and structure of the engineering profession, the practice of engineering, and the marketing of engineering services. Without plunging into a deeper discussion by defining education, few professors will award any considerable educational value to such subject matter, by itself. The engineering societies have authorized student chapters for the very purposes indicated—that is, to hasten the student's conception of the breadth of his profession, to bring the activities of the profession and its members to his doorstep, and to place in his very hands the professional literature available. However, some students never attend a meeting of their student group just as some engineers never attend a meeting of their local section, some physicians never attend a meeting of the medical society, and some attorneys never attend a meeting of the bar association. Incidentally, such men are not likely to attend class reunions in great numbers. That is personality. Its roots are congenital, hereditary, or fed by earlier environment. The way to improve the professional personality is to nurture the promising and eliminate the hopeless. Do not imagine that a college professor can be influenced to change an engineering curriculum for the purpose of raising the personality standard of his student body.

It has been stated that every man should be a salesman—that, if he has nothing else to sell, he has to sell himself. This is only partly true. Many of the most successful engineers never seek an engagement; engagements come to them. The same is true of the other professions. Being a broad and progressive profession, engineering has opened a field of technical salesmanship to introduce new techniques and new equipment in a conservative and stable society. Engineering salesmanship has been and will continue to be apparently more remunerative in the earlier years than design, construction, research, or production. Large firms have sales or contracting organizations, and technically trained young men who have a flare for that type of work seem to be in demand. Salesmanship, however, is almost as technical as surveying or thermodynamics and, instead of being founded on the immutable principles of mathematics, is based on human nature with all its vanity, superstition, and dogma. This field involves "how to make friends and influence people" and successful salesmanship requires much more study of human nature and self-discipline than most people realize. In the past, professional engagement has been based largely on record and recommendation, and has been continued because of performance. Agreeing that Alexander Pope was correct when he

[10] "Freedom to 'Talk Back,'" by T. R. Agg, *Civil Engineering*, September, 1940, p. 549

wrote "the proper study of mankind is man" and that popularity among one's acquaintances and geniality among strangers are desirable personal assets worthy of development, then, is or is not the engineering curriculum the proper place for this training (not education)? If "yes" is the answer to the question, it should be decided what subjects are to be removed from curricula to provide the necessary time and someone must accept the responsibility for the decision.

Mr. Baker states (under the heading, "Introduction") "Four years is too short a time," a fact not new to educators or engineers. From the purely professional standpoint, a longer course offers the solution to most of the problems. At the beginning of the century a number of universities offered longer courses; some still do, with a very small registration. Such courses are available, now, for the student who chooses them and, as they become more popular, the supply will probably meet the demand. The New York Section of the National Society of Professional Engineers has taken action on this subject and such action on the part of a few more such societies might force a five-year or six-year course. This trend may develop much faster than is realized. Thus, the effect on the student, the profession, and the university, somewhat interlaced, may be considered briefly.

The time and cost will increase and, presumably, result in fewer alumni. That is what happened in the medical profession. The next generation will have fewer engineers, but better trained ones. The demand for services will net the profession a larger slice of the fruits of the individual engineer's effort—an attractive prospect to the student who has the funds and is willing to devote the time required. It appears somewhat as though it would be a cog in a program to "make the rich richer and the poor poorer."

From the professional standpoint, most engineers are ready to endorse a program that promises to raise the standing of the profession. Those who have given time and thought to the subject feel that more time is the only solution and that the social problem can be met. Published data indicate that engineering is already a tolerably well-paid profession. It may be that engineers are already taking from society in a closer proportion to what they put in than is realized. Any movement that will result in the ability of the profession to contribute more to society will entitle the profession to a proportionally greater reward.

Engineering educators are likely to point to the first half of the twentieth century, with justifiable pride, as the era of most elaborate technical development in the history of human culture. This development has demanded and used both quality and quantity of effort. It has been predicted that, to maintain the present acceleration in development, society will require engineering talent of an increasingly higher quality in successive generations. On the other hand, it has not been predicted that it will require less; the reverse is more likely. The truly professional man would suffer from insomnia after participating in a plan to increase his emoluments by decreasing service. So it may be unwise to enter on a program likely to decrease the number of professional engineers without plans to supply the demand for a subprofessional group.

Many of the largest universities, supported by public funds, are enjoying mass educational programs developed on the basis of giving the greatest value

to the greatest number. Hoards of students have crowded the classrooms and laboratories of these institutions for technical educations in four years although registration remains pitifully low at institutions offering longer courses. The general result has been that men with vision, foresight, energy, and determination have forged ahead into places of responsibility whereas those less abundantly endowed with personal qualities have remained in the subprofessional group longer. This program has placed engineering in its current position.

Now suppose the profession should force a six-year course. The student would probably register in the arts course, elect the first two years of an engineering curriculum during his last two years, stay two years longer, and be graduated. Such a procedure presents no unsurmountable curricular problem. There will be fewer alumni but the university budget could probably be balanced by an increase in tuition. The supply of talent will be better qualified but smaller. The quantity will have to come from elsewhere. The most likely sources are the technical high schools or a forced development of technical junior colleges. A number of these are functioning in the larger cities of the United States. The popular four-year course will be unjustified in its present form. Concurrent operation of both the longer course and the shorter course on the same campus has not proved satisfactory. Universities with adequate equipment and staff will likely follow the signpost to the longer course and adapt themselves. Industry, already short of technical talent, will have to endure through the transmission years. Will it? Can it? That is a personnel problem.

The 1945 report of the Society's Professional Committee on Engineering Education [11] indicates that the Society is likely to adhere to the four-year course in substantially its existing form for the present and suggests that at least 20% of the students' time be devoted to humanistic-cultural subjects. What can or should be placed in this 20%?

The standard unit of credit, or "credit hour," is one hour of class per week per semester, requiring preparation—an average of two hours of preparation. Art courses usually require 120 credit hours for graduation, 15 per semester or five courses meeting three times per week. Three hours of laboratory work per week is usually awarded one credit hour. Engineering courses are proverbially longer, requiring from 132 to 144 credit hours for graduation—an average of 16 to 18 per semester and five or sometimes six subjects. After World War I, most of the short courses in such subjects as masonry, tunneling, descriptive geometry, stereotomy, railroad location, railway track and trackwork, and modern languages were eliminated, grouped, or taught with some other subject. In this way room was made for electives in the humanistic-cultural field. At present, the required courses comprise about 80% of the time and electives about 20%; 20% of 132 hours is 26.4 credit hours, or just about one standard three-hour elective throughout the entire four years. Assume that a student has a patriotic urge and wants to elect a course in the Reserve Officers' Training Corps so that he can become a reserve officer in the army or navy upon graduation. That activity absorbs all his electives. The

[11] *Proceedings*, ASCE, March, 1946, p. 386.

variety of subjects that a class of students desires to elect embraces almost all that can be offered. A fairly large number have visions of travel and elect a modern language, the subject which requires more of the student's time and from which he drives less benefit than any subject which is likely to be avoidable. Nevertheless, if electives are offered, they should be free electives. Much can be stated in favor of a rigid curriculum; it is good training to form the habit of doing what has to be done.

A "letter to the editor" by Don Johnstone,[12] Assoc. M. ASCE, includes some diagrams that show very well how a curriculum is arranged with the fundamental subjects first, then the techniques based on those fundamentals, and finally a smattering of design. The present work requiring 80% of the four-year course will comprise 64% of a five-year course or 53% of a six-year course. The five-year course would permit time to cover some of the most glaring deficiencies that Mr. Baker has indicated but the six-year course would give much more latitude.

Many institutions have courses required for every student. Such courses are usually broadening and have the virtue of being taught by inspiring professors. Every course or subject should have two elements—education and training. Even purely engineering subjects are presented in such a manner that the student must derive some education from the course as a whole, even if it is primarily intended as a vocational subject. If the professor fails in this objective, he is a failure and his students are merely trained in a prospective vocation. Much pressure has been brought on the colleges to insert almost entirely informational courses. Most technical courses are sufficiently informational to keep the memory pliable. Faculties have resisted suggestions for subjects that can be learned by postgraduate reading and have insisted on subject matter that requires teaching rather than merely reading—factual and descriptive teaching. The former practice of making an assignment in a textbook and hearing the lesson is not nearly so prevalent as it was a generation ago.

For present purposes, subject matter may be considered as (1) distinctly intended for use in earning money (vocational); and (2) for the student to live with during his entire life (cultural). The educator has felt that it is fully as important to show the student the importance of living a full and fruitful life as it is to teach him how to earn some money. There is little time for such work in a standard four-year engineering course. At the same time, the professors who recommend their progeny to prospective employers, and the universities they represent, would feel it very keenly if one of their former students failed to meet the requirement on the first task to which he should be assigned by his employer. It is much more likely that the young engineer's first assignment will be routine in type rather than a monumental design or even an economic investigation. Therefore, professors take much care to assure themselves that, with little or no assistance, the student can make a presentable drawing, run a line of levels, make readable field notes, and follow specifications and directions. They will probably continue this practice even in the face of criticism.

Criticism has been directed at surveying and drawing. Most universities devote about 6% of the students' time to the former and 4% to the latter.

[12] *Civil Engineering*, June, 1946, p. 267.

Surveying is the first course to which the student is subjected in which he is shown how to apply mathematics. Prior to this, mathematics has been something that his elders have forced upon him and which had to be mastered, perfunctorily in too many cases. If he cannot develop a flare for applied mathematics, he learns it quickly and changes his course. No attempt is made to create a supply of expert witnesses in court cases but the adept student will form some conception of the relativity of measurements, the limitations of accurate physical measurements, the inaccuracy of the arithmetical mean, the value of neat and accurate records (dated and witnessed), and the shape and movement of the earth and some of the heavenly bodies, as well as the construction, use, and adjustment of some measuring equipment. Such subjects are by no means purely vocational for it is a known fact that few engineers practice surveying or geodesy as a vocation. Much the same may be true of drawing. Few students know how to ink a right-line pen when they report in their first class. Some lettering and draftsmanship must be taught. If a little freehand sketching is included, it may supply the basis for a refreshing avocation—Leonardo da Vinci did very well. The time alloted to these subjects has been cut to the very bone already.

English has been and is the most usual subject of criticism; it has been a problem for years. Engineering is taught by engineers, medicine by physicians, law, even business law, by attorneys; but English—well—few instructors have succeeded in creating any considerable enthusiam in the student body. It certainly is as important a tool as some technical subjects, and has educational and cultural value. The scholarly instructor who is a leader and who fires the imagination with the beauty of metaphor, harmony, and clear concise exposition, is only too rare—so the boys use a formula to secure a grade. Consequently, the diction, choice of words, and clarity of expression of a senior show little improvement over those of a freshman. He knows more to say; he has a broader interest but his language discloses his early environment. This condition is not a matter of curricula; few engineering faculties can secure the solution; it is an institutional shortcoming that can be corrected only by the institution.

Graduate "refresher" courses have not been supported with enthusiasm. The formal part of a person's education usually ends with the valedictory. It is doubtful if this will change considerably; certainly it will not change quickly.

A rather important aspect of education has not been mentioned by Mr. Baker—graduates who do not follow engineering. The writer entered college with sixty freshmen, twenty six of whom were graduated (in civil engineering). Five years after graduation, only five were practicing their profession. A contribution of five per year to the profession by a large university is not great. Some sixteen hundred were graduated at another institution with the writer's approval. Of these, probably not more than four hundred practiced engineering more than a few years. The others are in every walk of life from politics to the ministry; they are even newspaper reporters and authors. Their careers justify their training and the existence of their alma mater.

To assist the student in the solution of his financial problem, colleges have appropriated funds for scholarships up to one third or more of the student body. A point that has not been solved satisfactorily is the creation of a motive, on the part of the student—a motive sufficiently strong to drive him over the goal. The postwar student body will consist largely of former service men. The expenses of the men have been provided for and few of them will require artificial motivation. They will have a standard four-year course with its virtues and its shortcomings. Industry and the coming generation will extend to them a hearty welcome, subject them to the usual hardship, and life will be the better as the result of their training.

L. E. GRINTER,[13] M. ASCE.[13a]—The sincere appreciation of the colleges of engineering should be extended to Mr. Baker for his thoughts on engineering education. He is more complimentary than the men who are responsible for the present form of the engineering curriculum would be. The singular "curriculum" is used with justification because the variations between schools and departments has never been great and the band of variations has been narrowed by accrediting procedures. One question to be considered is whether the author's concept that specialization is bad and that a broad general curriculum is to be preferred might not lead to further standardization without adequate justification.

It is fairly clear from published reports that during the past two years nearly all colleges and universities have been reconsidering their objectives, their methods, and the results obtained. In another year, most engineering curricula will have been rewritten. Educators have been taking very seriously the common criticisms that their graduates are poorly qualified to communicate their thoughts to persons in other groups and that engineers are almost antisocial in their public lives. Certainly it is clear that engineering does not attract young orators and perhaps infrequently enlists anyone who could earn his living through writing. Furthermore, the study of facts, theories, and more facts does not stimulate the development of either the communicative arts or the ability to make friends or influence the actions of others.

Engineering educators want to broaden their students, but few are convinced that a course in public speaking and another course in English literature, technical writing, history, or public affairs will accomplish the desired result. They believe that a good long list of such courses under cultured stimulating teachers might produce results, but they fail to see how one can study such subjects in sufficient quantity and still end up as a trained engineer. The writer adds that the result would be a total lack of education unless the engineering student could be made to want to study these nontechnical subjects.

Whether educators like to believe it or not, it is true that the student himself has a considerable influence upon the engineering curriculum. By simply doing poorly in courses that do not attract his interest, the student can eventually change their content or even eliminate them from the curriculum. Educational administrators would prefer to maintain common studies for all engineers

[13] Vice-Pres. and Dean, Graduate School, Illinois Inst. of Technology, Chicago, Ill.

[13a] Received July 1, 1946.

through the first two years of the curriculum. The student, by continually expressing his desire to start the study of some special branch of engineering immediately, has kept most institutions from adopting this eminently common-sense procedure. A uniform freshman course is as far as administrators have progressed against the desires of the student and of teacher specialists. Could it be that students and teachers are closer to the real purpose of education than the administrator plagued by schedules and budgets?

The question often arises as to whether the study of advanced technical problems in college has merit. In the words of the author (see heading "Prewar Engineering Education"), the student

> "* * * grappled with problems of a character that he would never meet, or at least would not have to solve, on his own responsibility for from at least twenty to twenty-five years after graduation."

Times are changing, or have already changed, to such extent that many laboratories are directed by engineers thirty years old or less. During World War II, young structural engineers carried grave responsibilities for airplane design work, and they will soon hold equivalent responsibilities in bridge and building design. There is no technical responsibility that a first-class engineer of thirty may not have an opportunity to assume. To this extent, the most advanced scientific training can be justified fully if it is not at the expense of a reasonably rounded education. The other objective of advanced scientific study is mental training in how to attack the analysis of such problems. It is not the particular problems that are important, but the experience of having solved difficult problems that develops self-confidence.

On the credit side of the engineer, the author rightly lists resourcefulness, persistence, and a willingness to accept hard work. Engineering education, as it has existed, must be credited with some of the development of these traits which are not common in liberal arts graduates. On the debit side are carelessness in the use of spoken and written English, lack of salesmanship, and an unawareness of the responsibility of the professional person for political and social relationships. The author wrongly criticizes the engineer for lack of a "dollar sense" which has long since become an inherent part of engineering education. In fact, no other group except business men shows so much interest in the practicality of its work.

The experience of counseling with young engineering students makes one skeptical of the practical results of attempting to force such individuals into the single pattern of broad general education of an engineering nature. The major institution that is famed for giving a single degree to all its engineering students still has been forced to provide considerable optional study despite tradition and the natural preselection of its students. It may be questioned, however, whether the number of vertical or professional subdivisions of the field that now exist are necessary or desirable. It is evident that the choice between mechanical and civil engineering, for example, should be made on the basis of interest rather than aptitude, and the same may be true for the various subdivisions of civil engineering. There are job classifications within the engineering field, however, which do require different kinds of aptitudes.

As an example, one may compare research and design on the one hand with operation and maintenance on the other. Another job classification clearly demanding a special group of aptitudes is management. As a contribution to the author's desire to educate some engineers broadly without missing the need that the writer sees for training others as specialists, the proposal of broad horizontal options through the entire group of engineering curricula deserves careful study.

One such option designed for the training of specialists might properly be in research, development, and design. Such an option would stress mathematics and its application to the solution of engineering problems. It would emphasize analysis and would also provide students with an opportunity for an internship of a few credit hours either in theoretical or laboratory research. At the opposite end of the educational spectrum would be the broad option in management which would provide for all the studies so aptly described by the author. In between would be professional engineering options, not essentially different from those in the present curricula, for students desiring to construct, operate, and maintain engineering works. These horizontal options are equally needed in all engineering fields.

It will become clear that the system of horizontal options provides an opportunity for the full use of the student's aptitudes within the field of engineering to which his interests lead him. He can still be a civil engineer while developing his talents for research, management, or practical engineering service. This is merely the theory of specialization applied in a new direction in engineering education. The result, if this concept is carried out at other institutions as it is planned at Illinois Institute in Chicago, will be to produce one limited group of highly trained specialists in research and the more scientific phases of design. Another limited group will be broadly educated with emphasis upon ultimate work in management within the field of civil engineering. No doubt this group will be more vocal—more interested in economics, politics, and social problems than have engineers of the past, but a larger group than either of these will still be trained for practical engineering work with considerable emphasis upon early competency for earning a living as civil engineers. If this is shortsighted, it is because practical young Americans will it so.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

EXPRESS HIGHWAY PLANNING IN METROPOLITAN AREAS

Discussion

BY HOMER M. HADLEY, DONALD M. BAKER, W. J. VAN LONDON, MERRILL D. KNIGHT, JR., AND R. H. BALDOCK

HOMER M. HADLEY,[8] Assoc. M. ASCE.[8a]—This is a truly admirable paper: Clear, lucid, and comprehensive. It covers the subject adequately in all major aspects without becoming involved in minor detail and trivialities, and the presentation is made so effectively and with such logic and reason that the stated best method of dealing with urban traffic congestion cannot fail to find acceptance in the minds of the great majority of readers.

In the development of detailed plans, arrangements for entering and leaving expressways and limited access highways offer the most troublesome problems. With two adjacent fast-moving streams of traffic in the same horizontal plane going in opposite directions, left-hand turns are neither permissible nor possible, and elaborate cloverleafs and grade separations may be necessary to deal with access and egress. This problem is not absent in elevated structures whose traffic, if all at one level, presents the same conflicts that ground traffic does.

In considering a specific case of this kind it appeared to the writer that these difficulties would largely disappear if a two-level structure were used with traffic separated—say, northbound on the upper level and southbound on the lower level. Under these conditions, access and egress are possible on either side; and, although special framing of the structure and special column positioning would be required at such places, these adjustments could readily be made. Moreover, a two-level structure actually would save right of way; for example, with a six-lane structure under the Design Standards for the National System of Interstate Highways,[9] a minimum width face to face of 83 ft of railings is required, whereas for a three-lane structure this dimension is 43 ft. A saving of a 40-ft right-of-way strip effects a very real saving in cost where land values are high.

NOTE.—This paper by Joseph Barnett was published in March, 1946, *Proceedings*. Discussion on this paper has appeared as follows: May, 1946, by Harry W. Lochner, and Fred Lavis.

[8] Cons. Engr., Seattle, Wash.

[8a] Received April 15, 1946.

[9] "Design Standards for the National System of Interstate Highways," A. A. S. H. O., adopted August 1, 1945.

How adversely such a two-level elevated structure would affect the use of a street beneath depends not a little upon the positions of columns. If they can be kept clear of the curb lines, the street traffic will not be affected at all; and, although the structure above would cause a certain loss of light, this loss would be far less than that beneath a structure of nearly double the width. If additional provision is made for a reasonable space between the viaduct structure and adjacent buildings and if the structure is given proper architectural treatment, the unfavorable effects would be slight indeed. Of course, there is the further consideration, stated by the author, that topography may make an elevated structure a necessity. When that is the case, the structure unquestionably can be made pleasing in appearance and any unfavorable aspects can be held to minor scale. When the Lake Washington Pontoon Bridge at Seattle, Wash., was first announced, its opponents declared not only that the beauty of the lake would be destroyed but that the value of the abutting property along the lake shores would also be destroyed. Nothing of the sort followed the construction of the bridge. The sole objectionable feature appears to be the noise caused by heavy trucks ascending the 5% grades on the elevated approaches. Passenger cars cause no annoyance whatsoever. In general, the bridge traffic is nonexistent, so far as ground level is concerned. True, this would not be the effect on tall buildings adjoining a viaduct. The traffic then would be at the level of upper floors. However, whether this coincidence would be objectionable or not depends on occupancy. In many cases it would be a matter of entire indifference.

The author makes no mention of vehicular tunnels in his paper. Obviously, they may be unavoidable at times and then they unquestionably are most acceptable. However, if their terminal points can be joined by other construction without too much loss of distance or grade, they appear to the writer to be the least desirable and most costly connections that can be considered. The author's views regarding vehicular tunnels from ½ mile to 1½ miles long are requested.

DONALD M. BAKER,[10] M. ASCE.[10a]—Express highways are the third factor in the development of trafficways in metropolitan areas. Streets in these areas were not a great problem until the introduction of the motor vehicle. They served adjacent property and were financed and maintained by local taxes. Early motor vehicle traffic, traveling at speeds several times those of horse-drawn vehicles, required better roadway surfacing, which was paid for by local taxes or assessments, and in some instances by local bond issues.

Increasing speeds and traffic density during the decade 1920–1930 led to the development of arterial highways, created by the opening, widening, straightening, and paving of certain principal streets upon which a large part of such traffic concentrated. These arterials served primarily the motor vehicles using them, rather than abutting and adjacent land; but they were financed in most instances by assessments upon this land, on the theory that the land was benefited by the improvement to an extent which was at least equal to, and usually in excess of, the cost of the improvement. This theory proved to be

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[10a] Received April 29, 1946.

sound in a few instances, but in widespread use was found to be a mistake, since the improvements benefited motor vehicle users far more than property owners, and the burden upon the latter was in many instances unsupportable.

Express highways should be planned and designed as mass transportation facilities to serve the people of a community; and to accommodate not only privately owned vehicles, but also public carriers, usually buses, although some situations will be found where they can accommodate rail facilities without too great an additional cost.

Until well into the 1920–1930 decade, mass transportation in urban and suburban areas was provided by rail lines. In most places these were located on the streets; but in the largest communities, where the capacity of surface lines was inadequate, they were constructed "offsurface." Offsurface rail facilities have greater passenger-carrying capacity per mile of track than do express highways per lane, and can serve more passengers per dollar invested in them than can express highways; but certain conditions have developed which, except under particular circumstances, make widespread extension of such lines very difficult of achievement.

The riding public does not appear to be willing to pay fares that will provide adequate service and yield operating expenses and debt service for offsurface rail facilities; this condition makes their financing primarily an obligation of the community itself. Demands upon public credit for the provision of other needed facilities, such as streets, schools, water and sewerage systems, parks and playgrounds, public buildings, etc., and the taxes necessary to pay for these facilities make the prospect of successful passage of public bond issues to pay for offsurface rail lines very discouraging.

Another drawback to their development arises from the decentralization of population in metropolitan areas, and the increasing ownership and utilization of motor vehicles. In the twenty-five years from 1916 to 1940, motor vehicle registration in the United States increased from 35 per 1,000 population to 236 per 1,000, or nearly seven times. With the revival of motor vehicle production, registration can shortly be expected to reach a density of one motor vehicle per family.

Prior to 1920, new population followed the extension of rail lines into undeveloped areas surrounding the cities, and settled in close proximity to these lines. People soon found, however, that by owning an automobile they could live anywhere within the area and travel back and forth to their place of employment, or make other travel movements without dependence upon the rail system. Ownership of automobiles was difficult in localities of high population density; and, since most people desired to possess one, this encouraged them to move to areas of lesser density. Congestion due to the increasing number of motor vehicles made travel by rail slower and increased operating costs, with the resulting decrease in service and also in patronage. This spiral has continued to the present time.

The construction of rural and intercity highways was initially financed by local taxation, and later by bond issues, until the motor vehicle fuel and use taxes were developed. These taxes were initially collected and administered by state authorities, whose jurisdiction seldom extended into the corporate

limits of cities; and more recently they have also been collected by the federal government for allocation to states. From the outset they have been devoted almost entirely to the construction and maintenance of roadways and highways outside of urban and suburban areas, although a high proportion was collected from motor vehicle owners and users operating largely within these areas. Beginning with the depression (1930–1940), improvement of streets and highways in heavily populated areas by bond issues and assessments upon the adjacent property proved to be infeasible; local pressure on state authorities forced the allocation of some of the proceeds of fuel and use taxes to cities and counties, although the amount so provided was seldom sufficient to more than care for maintenance and a nominal amount of arterial development.

At present, the network of rural and intercity highways is far more adequate to meet the needs of the traffic using them than are the highways and streets in metropolitan areas, with by far the greater vehicle mileage, at least in states with a high ratio of urban to rural population, occurring in cities and metropolitan areas. Tremendous financial losses resulting from deaths and accidents, delays due to traffic congestion, and losses of property values, occur in these areas. Gasoline rationing during World War II reduced the acuteness of the situation somewhat; but, even with the reduction in the number of motor vehicles since 1941, the elimination of gasoline rationing and the subsequent greatly increased use of automobiles are bringing about a serious situation. When renewed production of motor cars has brought registration back to prewar figures, this situation promises to be extremely critical.

Until recent years, the major interest of state and federal highway authorities, in so far as it applied to cities and metropolitan areas, appears to have been in providing facilities for by-passing traffic originating outside of these areas around their congested portions. Recently, however, there has developed a recognition of the needs of such areas for facilities which will reduce congestion within their boundaries.

The author is correct in his statement that the greatest need for express highway facilities is along routes radiating outward from the central business districts of metropolitan areas. A large volume of traffic does occur in a circumferential direction; but congestion comparable to that occurring on radial routes seldom occurs, since a far greater number of circumferential trafficways are usually available. Increasing congestion on radial routes as they approach central districts is due not only to the fewer number of trafficways existing as the central district is approached, but also to the steady contribution of traffic along these routes as they approach the central district which increases its total volume, and to the increasing interference by cross traffic at the more frequent number of intersections. In studies made as consultant to the Los Angeles Transportation Engineering Board in 1938–1939, the writer found that during the 12-hr period, 7:00 a.m. to 7:00 p.m., 35.7% of the total number of motor vehicles entering the central district of Los Angeles (Calif.) originated within a 2.5-mile radius of the central district, 53.1% originated within a 5-mile radius, and 63.7% within a 7.5-mile radius. Only 14.5% of those entering were registered from locations outside the metropolitan area.

The large use of motor vehicles, as a means of travel to the central district, is shown by the fact that 22.9% of the vehicles registered within the Los Angeles metropolitan area entered the central district during the aforementioned 12-hr period. Total persons entering the district during the 12-hr period by means of private automobiles equaled 15.6%, and by public transportation 10.7%, or a total of 26.3%, of the population of the Los Angeles metropolitan district. This may be compared with a figure found by the writer to be equal to 30% of the population of the metropolitan district entering the central district of Los Angeles in the year 1931. These percentages are consistent with the results of cordon counts around the central districts of ten large cities made during the late 1920's and the early 1930's, all such counts indicating that a number of persons equaling from 25% to 30% of the population of metropolitan areas enter the central district of such areas daily by various modes of transportation.

The need of by-passing traffic around central districts to relieve congestion in the approaches to and also within these districts, which was stressed by the author, is likewise acute. The suggested inner and outer by-passes are frequently desirable as an aid in the reduction of congestion in approaches to the district. In the foregoing studies, made in 1938–1939, it was found that from 25% to 40% of the vehicles entering the central district of Los Angeles were destined to points beyond it, and passed through it rather than followed existing but more circuitious by-pass routes. The elimination of this large number of vehicles from the streets within the area and from those approaching it would have done much to relieve congestion.

During the economic depression of the 1930's, many studies were made of traffic habits and characteristics within metropolitan areas, and many plans for relief of congestion were formulated. Most of these are still merely lines on paper and printed pages of text in reports. Difficulties in activating such plans always lie in finding ways and means of financing the improvements which they recommend. It is the writer's considered opinion that plans for express highways, or in fact for any other improvements, are too often wasted effort unless they include, in addition to the physical plan, a feasible one for financing the improvements recommended.

The "pay as you go" plan which contemplated the use of proceeds of fuel and use taxes derived from motor vehicles for financing highways was far sounder than that of using proceeds of 40-year bond issues to build roads and highways which had a 20-year life, or of assessing adjacent property to pay improvement costs; but city and regional planning, and highway engineering have made great advances since World War I. New highways can now be planned and constructed with the assurance that, given adequate maintenance, they will not be worn out or obsolete for at least half a century. Moreover, the "pay as you go" plan, even if fuel and use taxes are substantially increased, will not provide funds for express highway facilities now needed in metropolitan areas at a rate sufficient to catch up with increasing needs. These needs have been unsatisfied and accumulating, not since the date of Pearl Harbor, but since the beginning of the economic depression in 1929, and are increasing at an almost geometric rate.

Financial losses, due to inadequate travel facilities in metropolitan districts and the resulting congestion, are of vast proportions. In 1933, in a report made to the Central Business District Association of Los Angeles upon a rail rapid transit system for the Los Angeles area, the writer estimated that monetary losses in time to drivers of automobiles, and operating losses to public carriers, because of congestion and traffic delays in traveling to and from the central business district of Los Angeles, amounted to $15,000,000 annually. These were intangible losses. More tangible ones, due to reduction in property values, lost sales in retail stores in the area, traffic deaths and accidents, were not included in this estimate.

If adequate relief is to be achieved, expenditures of a very large order must be made, and made within the next few years, rather than over the next several decades. Because fuel and use taxes, even in increased sums, will be inadequate, the alternative would appear to be the issuance of bonds and the use of the increased tax revenue for debt service on the bonds.

Present interest rates are extremely low, and will be held so until the federal debt is greatly reduced. At current rates an annual revenue of one dollar over a period of from 40 to 50 years will pay interest and repay principal on from $25 to $30 in bonds, and allow an adequate sum for maintenance and operation of express highway facilities.

The cost of increased fuel taxes to a motor vehicle owner would be nominal. The average passenger automobile uses from 500 gal to 700 gal of gasoline per year. An increased gasoline tax of 2¢ per gal would cost the average owner from $10 to $14 per year, and would support from $250 to $420 in bonds for express highway purposes in metropolitan districts. Although savings in gasoline and tire mileage, and in the wear and tear on motor vehicles, may not entirely offset these costs, other and additional savings of a more intangible character, such as reduction in deaths and accidents, stabilization of property values, and reduction in travel time, will undoubtedly do more than offset the increase. The adoption of such a policy in the next decade, together with allocations from present state and federal revenues should fairly well satisfy the most pressing traffic needs of the metropolitan areas of the United States.

It is but fair and proper that motor vehicle owners outside of metropolitan areas contribute to the cost of express highway systems within these areas. In the past a substantial proportion of outside highways has been constructed and maintained by contributions from vehicle users operating within metropolitan areas. These contributions have been far out of proportion to the use made of outside highways by the urban and suburban contributors. Outside vehicle users will probably make as much use of metropolitan express highways as do the metropolitan users of the outside highways built through their financial aid.

If planners and highway engineers will give the same attention to financing improvements that they do to planning and designing them, the people will probably be able to enjoy them within a reasonably adequate time.

W. J. VAN LONDON,[11] ESQ.[11a]—The purpose of urban expressways is to provide fast and safe mass transportation for people and commodities within

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[11a] Received June 21, 1946.

and across metropolitan areas. Mr. Barnett has presented a comprehensive discussion of this subject. Such transportation facilities are as much the responsibility of the state and federal governments as are any other arterial highways. The Post-War Highway Bill stipulates that a certain amount of the funds provided by the bill must be used in the development of expressways in urban areas. In Texas the amount is about $27,000,000 when matched equally with state funds—this provision being mandatory.

The Texas Highway Department is responsible for design and construction of urban expressways. All costs are paid from state and federal highway funds except the cost of right of way, which is paid by the respective cities. The general design is that adopted by the American Association of State Highway Officials and is the highest design standard ever adopted by that organization.

The thought seems to prevail that expressway highways are for the purpose of handling through highway traffic. This is far from true. A national survey of all towns in the United States with a population of more than 10,000 shows that only 10% of the traffic on highway routes in cities is through traffic. The remaining 90% is local to what might be termed the metropolitan area. The percentage of through and local traffic varies with the size of the city and the nature of city's enterprises. An interview survey made in 1939 of 35,000 vehicles conducted simultaneously on ten highways leading out of Houston disclosed that only 6% of all traffic was through highway traffic. An analysis of the first expressway proposed for Houston indicates that of 45,000 vehicles per day anticipated in 1948 about 1,000, or 2.2%, will be through highway traffic. Therefore, it appears that urban expressways are what the name implies—transportation facilities for urban traffic.

It is apparent that so-called by-pass or belt highways cannot be justified for through highway traffic alone. Also, the more directly routed expressways will more effectively handle the small volume of through highway traffic than would a by-pass highway.

Urban expressways do not generally follow existing heavily traveled streets or highway routes, principally because of high right-of-way cost through the developed areas adjacent to these improved streets. Also, these streets usually lead through the congested business area which it has been found best to by-pass a few blocks to avoid congesting business streets with traffic passing from one side of the metropolitan area to the other. This type of traffic is about one third of the total.

In general, the expressway projects will begin at a connection with one or more state highways at the edge of the urban area and pass near, or through, the central business area to a similar connection at the opposite edge of the urban area. From the beginning to a point near the business area they will generally consist of four or six divided freeway lanes, with property service roads as required. All railroads will be passed under or over. Minor streets will be closed and major streets will pass under or over, with adequate access roads to provide for interchange of traffic. Design speed on the freeway lanes is 50 miles per hr. Near the central business area the traffic will be spread to several existing streets which will be used for a distribution system. Prob-

ably two of these streets leading directly to the expressway section beyond the business area will be developed as one-way streets.

The value of freeways is often questioned, the contention being that proper traffic light control at street intersection is almost as good as a grade separation. A perfect light-controlled intersection will handle 42% of the traffic that a grade separation will handle, which is probably a sufficient answer to the question.

The construction cost of the expressways contemplated will range from $750,000 to $1,000,000 per mile. The right of way will cost from $50,000 to $100,000 per mile. Immediately the question arises—are the expressways worth the money? The only justification for a public improvement of this nature is its economic value. A time-travel analysis on an existing route and one proposed project in Houston which will carry 25,000 vehicles per day shows a time saving of 12 min between two common points. Assuming 15 miles per gal for gasoline consumption and a cost of 20¢ per gal, the fuel cost is 1¢ per min at 45 miles per hr—the anticipated expressway speed. Computing 365 days at 25,000 vehicles gives 9,125,000 vehicles per yr. A saving of 12¢ per vehicle would be $1,095,000. The section of expressway analyzed will cost between four and five million dollars. The saving in fuel cost alone will pay for the improvement in less than 5 years. If other cost factors are considered, such as drivers' and passengers' time, reduced wear on equipment, ownership cost, wages to operators of commercial vehicles, earning value of commercial vehicles, etc., and an estimate of 3¢ per min is applied, the saving to the public would pay for the improvement in less than 2 years.

The planning of an urban expressway system, the over-all design, and the detail design features to provide fast, safe, and economical mass transportation for persons and commodities require very careful consideration of many factors. The first step in expressway development is to determine where and what the traffic is now, and where and what it will probably be 25 years hence. A route is then selected and a more detailed study made as to the present and probable future development of the affected areas. From these studies the location of street-grade separations, traffic interchanges, the number of traffic lanes required, etc., are determined. Little time and effort is required to make this statement but many months of tabulation and study of reports and data obtained from the various utility companies and bus companies; census bureau reports; city planning commission reports; chamber of commerce reports; past, present, and probable future population; and business and industrial trends; etc., as well as actual traffic counts and interviews are necessary to determine the "load for which the expressway structure should be designed." From these data, traffic flow maps are developed to determine where and what kind of traffic facilities are required. The proposed expressways which are expected to be completed by 1949 are designed to handle 1957 traffic loads.

The structural design features will also be based on the load to be carried. Bridge designs provide for the maximum anticipated loads. Earth and pavement structures will be designed for the maximum anticipated loads with a safety factor of about 1.5. The maximum permanent bearing value of the respective soils will be developed and the maximum anticipated traffic loads will be properly distributed over the soil foundation structures through various

pavement structures. The types and designs of pavement structures will be governed largely by economic considerations. The cost of the earth structures, and of pavement structures, will be much higher than that for the usual structures of this kind; however, they will more nearly approach permanency and the cost will be justified by their long and uninterrupted service to large volumes of traffic.

To a considerable extent the foregoing is a specific application of the general principles set forth in this paper. The traffic analyses and the economic evaluation of specific cases in Houston afford conclusive evidence of the soundness of Mr. Barnett's presentation of these matters.

MERRILL D. KNIGHT, JR.,[12] ASSOC. M. ASCE.[12a]—A natural and rational approach to the problem of express highway planning is presented in this paper. One element in the location and the design of expressways not sufficiently covered is their relation to land use. Too often in the past the zoning plan for a community has not been given the consideration which its importance in the over-all development would seem to justify.

Although origin and destination surveys have become recognized as prime tools in expressway design, these reflect only an existing condition and must be correlated with present and future land use before the analysis will achieve its greatest value in the planning process. In the Savannah, Ga., study, origin and destination surveys were used with due consideration for industrial expansion. However, the procedure still requires some refinements. It is suggested that further investigations are desirable to determine if specific land uses, when located in respect to other specific land uses, may be used as guides to changes which may be anticipated in traffic flow as the metropolitan area develops.

In the use of zoning maps in a location study, it is believed inadvisable to give too much importance to the consideration of expressways as barriers between zones of incompatible uses. Such a factor as an influence in design has been suggested by a number of engineers, notably, by Jac L. Gubbels.[13] Although expressways may assist in stabilizing values through the segregation of areas with comparable restrictions, this is certainly not their primary function. The primary function of moving traffic between areas of major generators immediately suggests that there should be a very real relationship between the highway plan and the land-use plan. To present a complete analysis of this relationship would require setting values on classified land uses as potential traffic generators.

Probably the greatest obstacle to such an analysis lies in the basis on which many zoning ordinances have been created. A typical city will undertake zoning primarily to protect property values. The first map to be drawn will be one showing existing uses. If retail businesses are widely scattered, the value of adjoining properties has already been depreciated for residential purposes. To avoid hardships resulting in the creation of districts containing too

[12] Highway Engr., Public Roads Administration, Washington, D. C.

[12a] Received June 26, 1946.

[13] "Planning for City Highway Traffic," by Jac L. Gubbels, *Better Roads*, October, 1945, p. 29.

many nonconforming uses, the tendency is to allow larger areas with fewer restrictions. This variation from the optimum allocation of land to classified uses admittedly interposes difficulties in estimating future traffic to be generated by areas presumably planned for specified uses. Such difficulties would be inherent in any city plan possessing sufficient flexibility to permit future growth. To make a plan effective all components should be developed toward a common objective.

Frank H. Malley[14] states that "* * * as cities grow the downtown area remains relatively constant in size as compared with the urban area." He believes that "* * * the proper location and design of Urban Freeways is the greatest single element in the cure of cities' ills and in the directing of their proper and adequate future growth." The determination of the pattern for future urban growth constitutes the most complex phase of selecting locations for proposed expressways.

The Regional Planning Board for the County of Los Angeles (Calif.) in a discussion of the "Master Plan for Highways, 1941"[15] explored, on an over-all basis, the highway requirements as related to the various land uses within the entire area. To do this a pattern of land use was developed, toward which it was believed the ultimate growth of the region should be directed. However, the pattern fixed as a goal for Los Angeles was based on a desire to avoid an overcentralization which has already become a fact in other large metropolitan areas. Hence, it cannot be used as a pattern for many other places. In the 1941 plan, Los Angeles expressways were proposed to serve more as intercommunity highways than as a means of moving people from outlying areas to one or two congested centers. Other cities can benefit from a study of the Los Angeles approach, even though it is not entirely applicable to their problems.

Keeping in mind the necessity for the transportation of workers between their homes and places of employment, it is important to stress that, in assembling basic data, an analysis of land use, both existing and potential, when properly correlated with origin and destination surveys, can be of the highest value in selecting the route which will best meet present and prospective needs. To increase the significance of such analyses, however, further study is required before making any attempt to assign empirical values to specific land uses as traffic generators.

R. H. BALDOCK,[16] ESQ.[16a]—A remarkably clear explanation of the present need for urban traffic facilities permitting uninterrupted flow and a proper relating of these facilities to local service streets and terminal facilities are given in this paper. Mr. Barnett mentions the ever-increasing realization that intelligent planning must rely on factual information in regard to the needs and desires of the populace. These factual bases can best be obtained through the medium of comprehensive origin and destination studies, the techniques of which are outlined briefly. Indeed, the data assembled can be used in an

[14] "Location and Function of Urban Freeways," by Frank H. Malley, Postwar Patterns of City Growth, Am. Transit Assn., p. 18.

[15] "Master Plan for Highways, 1941," Regional Planning Board, County of Los Angeles, Los Angeles, Calif., 1941.

[16] State Highway Engr., Salem, Ore.

[16a] Received July 10, 1946.

infinite number of ways to solve the many and varied traffic problems in a metropolitan city. Mr. Barnett makes an interesting statement to the effect that the development of a pattern of arterial routes results in a remarkably uniform solution for cities of comparable size.

The writer is in full accord with the thoughts expressed in the paper. However, he takes the occasion to stress the following conditions:

1. The current pressing need for adequate terminal facilities to provide storage for varying periods of time, particularly in and near the central business district.
2. The need for improved and adequate mass transportation facilities to attract a greater proportion of people.
3. The job of selling a bold and forthright plan of express highways and terminal facilities to the local citizenry through detailing and publicizing the economic benefits that will accrue.

The writer has stressed the matter of terminal facilities not because this phase should necessarily carry a disproportionate weight in an over-all program but rather because deliberations and planning with respect to express highways have far outstripped planning for terminal facilities. Notwithstanding the fact that there is still much to be done in the matter of planning arterial expressways, it would seem that more detailed study should be accorded the problem of terminal facilities to bring it abreast, so to speak, of progress in the matter of arterial development. The construction of expressways will divert and consolidate a large part of traffic on parallel streets between major points of origin and destination. In addition, it will generate a new and additional increment of traffic by the very reason of the improved facility. This latter increment of traffic will operate to complicate the present acute parking situation even further.

Moreover, an extended period of time will be required to build all the express highways and circumferential routes in a given metropolitan area and the present critical parking problem cannot wait. The terminal parking problem needs to be solved as part of the completed plan so far as feasible. It is further obvious that private enterprise cannot very well conduct the requisite preliminary studies for an intelligent appraisal of the parking problem.

In general, parking locations should be widely dispersed in and around the business district to prevent congestion on adjacent streets and to serve the public better. It may be found necessary for the city to acquire property for development of off-street parking facilities through the power of condemnation, if necessary, and to operate such facilities through leases to private individuals. Real estate values, and the economic requisite of realizing a fair return on the investment, may necessitate building a one-story or two-story structure to increase the parking space. The upper floor or floors can be made accessible by ramps and, where topography permits, by entrances from two different streets at different levels. In other cases, it may be desirable either to build or to give a long-period lease for the erection of a modern office building—the interior of which is utilized either through ramps or through elevators for the parking of motor vehicles. Soundproof and fireproof construction can be used;

and, if the architecture is attractive, the utilization of the core of the building for car storage should not detract from the value of the office space around the exterior of the building. To accomplish these improvements, it may be necessary, by state legislation and city ordinances, to establish a parking authority and to provide funds for the capital investment and the acquisition of sites to be amortized through rentals over a period of years.

Mr. Barnett's statement that the truck-loading problem will be solved only through a bold approach is correct. Truck-terminal and loading-dock space inside the building line of the ultimate destination of goods will eliminate first-floor street frontage for truck unloading—an important observation. The incorporation in city ordinances of off-street parking and loading facilities as a condition precedent to new building construction is still in the pioneering stage; however, such a measure appears to be the only permanent solution of this aggravating problem.

Turning to mass transportation and its bearing on the problem, Mr. Barnett states (under the heading, "Planning Agencies"):

> "If the major highways are developed as free-flowing expressways, the tempo of transportation in the city can be increased with favorable results for all. The writer cannot agree with visionary enthusiasts who believe that freeing transportation will of itself rehabilitate a city, although it can be of material aid. The forces of decentralization are too great to be stopped entirely, but the proper location and development of expressways can assist in the establishment of self-contained stable neighborhoods and in the stabilization of trade and values in the principal or central business district."

The increase in the use of automobiles by people working or trading in the central business district cannot proceed to the point where every movement takes place through the use of a personal conveyance. It may be impossible to build the requisite expressways, and certainly it is impossible to construct the required terminal facilities to care for a greatly expanded use of private automobiles—nor is such use economical.

There appears to be a demand for greater use of mass-transportation facilities. The building of expressways and properly located bus terminals will facilitate this trend; and it will give to the user greater safety and almost equivalent speed and comfort of movement. Express and skip-stop bus service can be inaugurated on expressways from suburban districts into the heart of the city. The modern city planner should give careful consideration to this phase of transportation inasmuch as it appears to offer an excellent solution to the pressing problem of traffic congestion in regard to both movement and vehicle storage. If a proper solution is reached, many citizens using their own personal automobiles between the suburbs and the central business district will travel by mass transportation, for convenience and economy. It may, of course, be necessary to use the automobiles from the home to a suburban bus terminal. The lower land values, in such cases, should make possible car storage at reasonable rates.

In the selection of the type of mass-transportation vehicle, the street car running on rails cannot be used because of its inability to conform to the patterns of the traffic stream. Likewise, although the trolley bus is more flexible

than the street car, its range of movement is definitely limited, and at times it becomes an obstruction to the free flow of traffic. The expense of erecting and maintaining the overhead wiring system is material and this type of construction detracts from the beauty of a well-designed expressway. With adequately powered motor buses, discomfort can be materially lessened while the freedom of movement makes for ideal operation in a stream of other motor vehicles.

Turning to the job of "selling" the local citizenry on the economic and social benefits accruing from building express highways and providing for adequate terminal facilities, Mr. Barnett has stated that the preliminary engineering report should describe fully the reasons for the expenditures proposed. Although many of the benefits accruing to the public by reason of the building of adequate express highways and terminal facilities are intangible and difficult to evaluate, in most instances it is possible to show that the yearly monetary benefits are much greater than the annual cost of amortizing and maintaining the facilities.

The annual road-user benefits, represented by the saving in cost of motor vehicle operation, the saving in time, and the saving by accident reduction, can be determined. When expressways have been planned carefully, based on origin and destination studies, the amount of these benefits accruing to a large volume of traffic is surprising.

As Mr. Barnett has stated, the direct road-user benefits are not the only returns. Intangible benefits which reduce driving strain in many cases far outweigh in the public mind the monetary values which accrue. An economic analysis properly presented to progressive civic groups, indicating the direct returns on the investment as well as the many indirect benefits which materialize, should sell such a program even to the more skeptical. In the matter of "selling" the program to a civic group, it is probable that engineers working on the technical phases are somewhat reticent to dramatize supporting data, and thereby fall somewhat short of making a case that will appeal to, and interest, a lay group.

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DISCUSSIONS

LANDSLIDE INVESTIGATION AND CORRECTION

Discussion

BY JACOB FELD, GEORGE S. HARMAN, AND ROBERT F. LEGGET

JACOB FELD,[5] M. ASCE.[5a]—The landslides reported in this paper are spectacular demonstrations of adjustments often occurring within soil bodies because of load changes or modifications of physical conditions. The internal adjustments result from (*a*) an increase in either static or dynamic loading externally, (*b*) a decrease in a lateral resistance or support, or (*c*) a modification of the soil constituents causing a decrease in physical resistance characteristics. Such internal adjustments very often do not exhibit any exterior result. When an exterior displacement becomes visible, it is called a landslide. However, the same phenomenon is encountered in engineering work and treatments similar to those described by the author become necessary even for cases which are not designated as landslides. For instance, when the fills in back of retaining walls or abutments settle because of either changes in external load or changes in internal characteristics, when structures and pavements along the side of railroad cuts and subway operations move either laterally or vertically, and when buildings constructed on hillsides or on filled-in ground covering sloping original surfaces show sign of distress—all these phenomena are landslides.

In general there are two types of correction: (*a*) The physical lateral resistance or support to a mass of soil can be increased by adding a retaining wall, sheet piling, or other structural additions; (*b*) further change of the physical make-up of the soil can be prevented by control of internal water, or by chemical injections, or by other methods that have been developed to alter the property of weak soils. For a number of cases in which building movements resulting from internal soil failures occurred, the methods of correction were published by the writer elsewhere.[6]

No general rules can be devised for sufficient and necessary methods of altering the physical characteristics of soils. Before any decision on such work can be reached, it is necessary to obtain complete information about the soils

NOTE.—This paper by Hyde Forbes was published in February, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1946, by T. W. Lambe.

[5] Cons. Engr., New York, N. Y.

[5a] Received June 5, 1946.

[6] "Several Foundation Repair Jobs," *Proceedings*, Brooklyn Engrs. Club, April, 1930, p. 47.

that exist in the locality; to know how these soils react with different moisture conditions under varying pressure and varying temperature; and to realize the effect of local chemicals that may be brought into the picture by the soil moisture. It is known that different chemical constituents in clays can be recognized and that the success of chemical control of clays depends upon the chemical constituents in the clays.

The effect of added absorbed moisture is described by the author (under the heading, "Landslides") as a weakening of the soil particles so that they cannot support loads. This is not always the case, as is exemplified in the description under the heading, "Shear Slides: Plastic Material at Market and Glendale Streets, San Francisco." The hydration of some soils retained within the materials which are not affected by water may increase the bearing capacity of the mass. This condition occurs when a formation swells from the absorption of water and integrates with adjacent layers. Some of these mixtures are similar to hardpans in their resistance to excavation.

Filling up cracks in clays overlying rock or other unalterable surfaces with rain water (under the heading, "Shear Slides: Earth Slide at Crolona Heights in Crockett, Calif.") is identical with the cause of failure of basement walls built in clay areas if surface waters are not properly collected. In both the Hartford, Conn., and Detroit, Mich., areas, the writer has seen basement walls pushed in by the pressure of water filling in a narrow but continuous crack when clay backfill dried out and left a gap along the exterior face of the wall. The use of underground stone drains in soils of this nature is only a temporary correction, since the clay soils find their way into the stone and seal up the ground. The only solution is to eliminate the water.

In attempting to prophesy where slides, either of the spectacular type or of the equally troublesome internal adjustment type, may occur, it is necessary to have a complete picture of subsurface ground conditions. Trouble will almost always occur in areas where natural drainage channels have been buried. Near Newburgh, N. Y., where some fills almost 60 ft deep had been placed under proper supervision and by heavy equipment, an internal adjustment occurred quite suddenly, forming a crack similar to that resulting from earthquakes. Fortunately, the crack developed one day before a concrete pavement was placed on a prepared subgrade which had been tested for bearing capacity and had been approved. Borings taken of the soil adjacent to the point of failure indicated that an old drainage channel at the bottom of the fill was still in operation and that the material at a depth of 60 ft was too soft to obtain any kind of sample.

A reliable method of determining the nature, thicknesses, and direction of the slope of the subsurface layers of soil is by means of test pits. The cost of such procedure is much more than the cost of borings and the accuracy of the information obtained is proportionate to the cost of the two methods. In connection with the methods of determining subsurface conditions, it is difficult to agree with the author in his statement (under "Shear Slides: Costs of Corrective Works") that boring records and samples are of little value if they are not collected by an experienced geologist. In some soil locations, boring records are of little value regardless of who collects them. The presence of an

experienced geologist does not increase the value of the soil sample which has been improperly taken. The writer has found a collective history of the vicinity and especially a comparison of maps showing topographical conditions over a period of years to be of great value. Such information has permitted the design of necessary corrective work for several subway operations in New York, N. Y., since the data indicated the direction and, to some extent, the amount of movement over a period of years. In this way artificial resistance can be provided to prevent incipient movement of adjacent structures. For instance, the record of surface elevations in the vicinity of 8th Avenue and the Harlem River in Manhattan (an area adjacent to the Polo Grounds) was a history of slowly decreasing movements in certain directions. Later excavations and test pits as well as borings provided sufficient information to prepare comprehensive subsurface maps indicating the slopes of the various layers. The movement predicted by this study agreed almost exactly with that found from the previous collection of data—on movement of monuments, amount of settlement of adjacent structures built at different times, and movements of streets. Movement of streets is not an unusual condition in cities built along waterfronts where almost always the marginal streets are on filled-in ground. In such locations corrective methods are often necessary and the methods used by the author in the correction of landslides in the San Francisco Bay vicinity have yielded very valuable data.

GEORGE S. HARMAN,[7] ASSOC. M. ASCE.[7a]—The Recreation Commission of San Francisco, Calif., from time to time has acquired parcels of land which, because of topography, instability, or both, are not particularly adapted to commercial or residential development. However, playgrounds, which supply a space for children to play in an otherwise highly congested district, can be built on these lands.

In designing a playground, the recreation profession and recreation leaders desire areas as large and with as few levels and terraces as possible. Thus, since San Francisco is a very hilly city, some heavy cuts and fills are necessary when grading a parcel of land for playground purposes.

When it became incumbent on the writer to determine engineering policy for developing such lands into playgrounds, it soon became evident that special knowledge in geology and experience in underground water were required. This need was filled when Mr. Forbes was employed as a consultant; and, since the writer followed the investigation and construction phases closely, he is most pleased that the unique conditions and methods used to solve the problem at St. Mary's Playground have been presented to the Society as a whole.

One of the principal benefits to be derived from the section on "St. Mary's Playground Slide" is the very practical approach to the solution of a problem of engineering geology in a hilly area with very complex soil and rock composition. The fact that investigation of the problem and remedial engineering construction work were combined in one project saved time, work, and money. The construction itself was quite novel in that it allowed the author to feel his way and to control the underground water as soon as it was encountered.

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[7a] Received June 17, 1946.

Many ideas were considered in trying to overcome the slide conditions, but all of them were discarded in favor of the method used because neither the depth of the underground water nor the location of the source were known.

Precise surveys and levels over a period of a year after the work on St. Mary's Playground have shown that the slide has been stabilized, whereas, previous to construction, each winter the slide was moving progressively many feet south.

The author mentions briefly the cause of the sliding at James D. Phelan Beach. Before this site can be used as a recreational area, the slide must be totally stabilized and a sea wall constructed to control the sandy beach and to prevent undercutting by high tides and wave action. As the author states, the underground water is causing the underlying serpentine rock to deteriorate and to disintegrate into a blue clay, thus producing a slip plane. This disintegration is not apparent on the surface, but borings and preliminary investigation have shown that this is the case. Sufficient remedial work has been done to prove that this slide can be stabilized by properly controlling the underground water. However, the partial installation also showed that time and money can be saved by having the advice of a consultant geologist during construction.

ROBERT F. LEGGET,[8] M. ASCE.[8a]—The description, presented by Mr. Forbes, of a number of landslides typical of the San Francisco area is a most valuable addition to the published records of earth movements affecting civil engineering works. Since, as the author rightly observes (under the heading, "Correction Should Be Based Upon a Thorough Investigation")—"Each type of slide and each set of conditions will present a particular problem * * * and no general treatment can be prescribed"—it is impractical to offer any useful general comment upon the individual cases included in the paper. However, it is especially significant that serpentine is apparently predominant among the rock types encountered locally, being referred to (under the heading, "Shear Slides: Parker Avenue Slide in San Francisco") as "the serpentine bedrock of the region" and later (under the heading, "Detrital Slides: O'Shaughnessy Boulevard Slides") as "basic igneous dikes, principally serpentine." Accordingly, it is puzzling to read the author's statement (also under the heading, "Detrital Slides: O'Shaughnessy Boulevard Slides") that "There is nothing unique about the rock formations of the San Francisco Bay area that makes them particularly susceptible to the development of landslides * * *." In the strictly literal sense, serpentine cannot be described as unique, but it is encountered as bedrock in built-up areas so infrequently that this statement seems to require some qualification.

Serpentine has long been recognized as one of the most treacherous of rocks, at least by the civil engineer. The author himself comments (under the heading, "Shear Slides: O'Shaughnessy Boulevard Slides") on "* * * the serpentine hydrates forming a clay which, with the further absorption of moisture, becomes a viscous mass moving down the slope * * *." (Presumably this is what is meant by the "landslide material" mentioned under the heading, "Shear Slides: Preventive and Precautionary Measures.") Very fortunately, there

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[8a] Received July 11, 1946.

are few if any other varieties of solid rock which possess the undesirable features of serpentine. Therefore, it may be suggested that some of the earth movements so ably corrected and described by Mr. Forbes were due, at least in some measure, to a special cause—the presence of serpentine.

If this suggestion is true, it is even more difficult to draw any general conclusions from this study of selected cases. Despite such a limitation the author has suggested a classification of earth movements which has little to commend it as compared with previously published classifications.[9,10] Only unusual local circumstances could be held responsible for having led the author to make statements in the "Introduction" which are not quite in agreement with a large body of informed opinion previously expressed and recorded.

The great value of the factual descriptions presented by the author make the submission of any critical comment an invidious task. However, for example (see heading, "Introduction") it would be singularly unfortunate if it were recorded in the publications of the Society, without refutation, that "* * * most of the laws and experimental data in this field [soil mechanics] have been developed through the use of dry sand composed of from 95% to 99% stable silica." The contents of the *Transactions* of the Society alone suffice to question this assertion. Although, it may be true (see heading, "Introduction") that, in the San Francisco Bay area, "Sand is seldom found disassociated from silt and clay, * * *" there are at least a few other parts of North America where even this statement is not quite correct. Other statements in the "Introduction" could be questioned, and there would surely be few engineers who would urge the use of slope stability calculations "as a substitute for * * * thorough preliminary investigation of all other determinate facts." Rather such calculations can only be used in conjunction with a thorough study of site conditions. There are many valuable publications which substantiate this suggestion, of which only two need be mentioned—that by A. W. Skempton in 1945[11] and that by Alex Collin in 1846.[12]

With Mr. Forbes' emphasis upon the importance of ground water in many cases of earth movement, there will be very general agreement. Not the least valuable part of the paper, therefore, is the statement of actual costs for the several types of drainage installations used by the author. The actual records of drainage discharge are similarly interesting and show in no uncertain manner the significance of ground-water movement at the slides in question. When plotted graphically, the discharges show somewhat erratic correlations with rainfall records; it would be interesting to know the author's explanation of this condition. Water can affect slope stability in ways other than those outlined by the author. Another paper, published in 1945, gives useful relevant illustrations.[13]

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[9] "Landslides and Related Phenomena," by C. F. Stewart Sharpe, Columbia Univ. Press., New York, N. Y., 1938.

[10] "Landslides, Subsidences and Rock-Falls, as Problems for the Railroad Engineer," by C. E. Ladd, *Proceedings*, A.R.E.A., Vol. 36, 1935, pp. 1091–1162.

[11] "A Slip in the West Bank of the Eau Brink Cut," by A. W. Skempton, *Journal*, Inst. C. E., Vol. 24, 1945, pp. 267–287.

[12] "Recherches Experimentales sur les Glissements Spontanes des Terrains Argileux," by Alex Collin, Paris, 1846.

[13] "The Stability of Natural Slopes," by W. H. Ward, *The Geographical Journal*, Vol. 105, 1945, pp. 170–177.